Equilibrium Behavior in Competing Dynamic Matching Markets

Zhuoshu Li, Neal Gupta, Sanmay Das, John P. Dickerson
Motivation: Kidney Exchange
Motivation: Kidney Exchange

Recipients

Wife

Brother
Motivation: Kidney Exchange
Motivation: Kidney Exchange

Husband

Wife

Donors

Recipient

Brother

Brother

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Motivation: Kidney Exchange

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Kidney Exchange is Dynamic
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- Patient-donor pairs (agents) arrive gradually over time
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  - stay in the market to find a compatible pair
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- Patient-donor pairs (agents) arrive gradually over time
  - stay in the market to find a compatible pair
  - may leave if the patient’s condition deteriorates to the point where kidney transplants become infeasible
Planner / Clearinghouse Platform
Planner / Clearinghouse Platform

• Minimizes the number of agents who *perish* (leave the exchange without finding a match)
Planner / Clearinghouse Platform

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• Knows agent’s expiration time\(^1\)

\(^1\) M. Akbarpour, S. Li, and S. O. Gharan. Dynamic matching market design. 2017
Planner / Clearinghouse Platform

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Planner / Clearinghouse Platform

• Minimizes the number of agents who *perish* (leave the exchange without finding a match)

• Knows agent’s expiration time[1]

• Has only probabilistic knowledge about future incoming agents

• Selects a subset of acceptable transactions at any point in time

Greedy and Patient Exchanges

Greedy algorithm

Patient algorithm

M. Akbarpour, S. Li, and S. O. Gharan. Dynamic matching market design. 2017
Greedy Exchange

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Two Questions: Strategic Agents

- Patient algorithm
- Greedy algorithm
Two Questions: Strategic Agents

Short-lived: $T_s$

Long-lived: $T_l$

$T_s < T_l$

$\theta$
Two Questions: Strategic Agents

Short-lived: $T_s$

Long-lived: $T_l$

Which market to enter?
How is the social welfare affected?

Greedy algorithm

Patient algorithm
Two Questions: Strategic Markets

Patient($\alpha^1$)

Patient($\alpha^2$)
Two Questions: Strategic Markets

Patient($\alpha^1$)

Patient($\alpha^2$)
Two Questions: Strategic Markets

Patient($\alpha^1$)

Patient($\alpha^2$)

$(1-\gamma_1)\gamma_2$
Two Questions: Strategic Markets

\[10\]

\[\text{Patient}(\alpha^1)\]

\[(1 - \gamma_1)\gamma_2\]

\[(1 - \gamma_1)(1 - \gamma_2)\]

\[\text{Patient}(\alpha^2)\]
Two Questions: Strategic Markets

\[
\text{Patient}(\alpha_1) = (1-\gamma_1)\gamma_2
\]

\[
\text{Patient}(\alpha_2) = (1-\gamma_1)(1-\gamma_2)
\]
Two Questions: Strategic Markets

How do interactions between overlapping pools, different matching rate affect social welfare?
# Utility Functions

<table>
<thead>
<tr>
<th>Agents</th>
<th>Markets</th>
<th>Number of matches</th>
</tr>
</thead>
</table>

![Graph showing utility functions for short-lived and long-lived agents over sojourn time.](image)

- **Utility Function**: Decreases over time, indicating diminishing returns as sojourn time increases.
- **Short-lived**
- **Long-lived**

**Legend**:
- Blue line: Long-lived agents
- Red line: Short-lived agents
Model I: Strategic Agents
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- Fixed matching policy: one is Greedy, the other is Patient
Model I: Strategic Agents

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• Random Agents: allow a $\phi$ fraction of random-choice agents
  
  - choose either market with 0.5 probability
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• Strategic Agents: $1 - \phi$, decide which market to enter upon arrival based on her expected utility
  - $\theta$: short-lived, $1 - \theta$: long-lived
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- Analyze equilibrium strategies of strategic agents given $\theta$ and $\phi$
Model I: Agent’s Tradeoff

- Matching probability vs utility
  - Patient market: higher matching probability due to market thickness, lower utility due to waiting
  - Greedy market: lower matching probability due to market thinness, higher utility due to immediate matching
Model I: Experimental Results

\( \phi = 0.4 \)

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<th>Pooling Equilibria:</th>
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<td></td>
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\( \theta \) values:

- 0
- 0.2
- 0.4
- 0.6
- 0.8
- 1
Model I: Experimental Results

\[ \phi = 0.4 \]

Pooling Equilibria:
- Short-lived: Greedy
- Long-lived: Greedy

Separating Equilibria:
- Short-lived: Patient
- Long-lived: Greedy

Pooling Equilibria:
- Short-lived: Patient
- Long-lived: Patient

Increasing proportion of short-lived agents
Model I: Experimental Results

\[ \phi = 0.4 \]

\[ \text{Expected utility} = 0.4 \]
Model II: Strategic Markets
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- Still a two-market system: Patient($\alpha_1$), Patient ($\alpha_2$)
Model II: Strategic Markets

- Still a two-market system: Patient($\alpha_1$), Patient ($\alpha_2$)
  
  - $\alpha$: parameter for the degree of patience (Akbarpour et al. 2017),
  
  higher $\alpha$ means more patient
Model II: Strategic Markets

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  - Agents stochastically enter either Market 1, Market 2 or both markets (Das et al. 2015)
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  - $\alpha$: parameter for the degree of patience (Akbarpour et al. 2017), higher $\alpha$ means more patient
  - Agents stochastically enter either Market 1, Market 2 or both markets (Das et al. 2015)
  - Markets respond to each other under best response dynamics. At any time period
    ‣ one observes the matching rate of its competitor
    ‣ chooses maximum payoff strategy for perpetuity for the next time period
Model II: Market’s Tradeoff

• Faster matching rate
  • Increased share of agents that enter both markets
  • Match fewer agents that only enter this Market
Model II: Experimental Results
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- Convergence to the Patient strategy under appropriate initial conditions $(\alpha^1, \alpha^2)$
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• For markets with sufficient overlap, and low initial values of \((\alpha^1, \alpha^2)\), convergence to a (Greedy, Greedy) equilibrium
Model II: Experimental Results

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• For markets with sufficient overlap, and low initial values of \((\alpha^1,\alpha^2)\), convergence to a (Greedy, Greedy) equilibrium

• No other phenomena occur in more than 5% of bootstrap samples
Policy Implications

• Significant social welfare losses through fragmentation

• Race to the bottom: Suboptimal matching policies
  - The United Network for Organ Sharing (UNOS) matches per month to now 2+ times per week due to competition with fast-matching the National Kidney Registry (NKR)

• Our model informs the debate