Incorporating Compatible Pairs in Kidney Exchange
A Dynamic Weighted Matching Model

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WUSTL Medicine: Jason Wellen
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Overview
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  • New algorithm —ODASSE based on online primal-dual
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• Matched 50% more incompatible pairs
• Increased expected graft survival by 1 - 2 years for compatible pairs
Living Donor Kidney Transplantation

- About 100,000 people waiting for kidney transplants in the US (2016)
- In 2014, 17,107 kidney transplants took place, ~ only 1/3 from living donors
- Unfortunately, willing living donors are often not medically compatible.
- One option for them is to enter a kidney exchange program
Kidney Exchange

Roth, Sönmez, and Ünver, 2004, 2005
Kidney Exchange

Roth, Sönmez, and Ünver, 2004, 2005
Kidney Exchange

Donors

Husband
Brother

Recipients

Wife
Brother

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• Better algorithm design
  ▶ Increase the number of transplants
    Abraham et al, 2007; Anderson et al 2015, 2017; Ashlagi et al 2015; Dickerson et al 2015, 2016; and many.
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• **Modeling matching quality**
  ▶ Consider to incorporate compatible pairs
  ▶ Provide incentive for compatible pairs to participate
**LKDI Score:**

9

This model calculates a risk score for a recipient of a potential live donor kidney.

**Live Donor Characteristics:**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
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<tbody>
<tr>
<td>Donor age:</td>
<td>43</td>
</tr>
<tr>
<td>Donor sex:</td>
<td>male</td>
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<tr>
<td>Recipient sex:</td>
<td>female</td>
</tr>
<tr>
<td>Donor eGFR:</td>
<td>95</td>
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<td>Donor SBP:</td>
<td>130</td>
</tr>
<tr>
<td>Donor BMI:</td>
<td>24</td>
</tr>
<tr>
<td>Donor is African-American:</td>
<td>No</td>
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<tr>
<td>Donor history of cigarette use:</td>
<td>No</td>
</tr>
<tr>
<td>Donor and recipient biologically related:</td>
<td>Yes</td>
</tr>
<tr>
<td>Donor and recipient are ABO incompatible:</td>
<td>No</td>
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<tr>
<td>Donor/Recipient Weight Ratio:</td>
<td>0.90 or higher</td>
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<td>Donor and recipient HLA-B mismatches:</td>
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Massie et al, A risk index for living donor kidney transplantation, 2016
Heterogeneity of Match Quality

Homogenous

Heterogeneous

Patients

Donors
Single Center Analysis
Single Center Analysis

- De-identified compatible pairs from 2014 - 2016
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- Counterfactual simulations: potential to improve outcomes
Single Center Analysis

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  - Optimal: arbitrary length cycles
Single Center Analysis

• De-identified compatible pairs from 2014 - 2016

• Counterfactual simulations: potential to improve outcomes
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  ▶ Two and Three-cycle swap

---

![Diagram of Recipients and Donors]
Single Center Analysis

• De-identified compatible pairs from 2014 - 2016

• Counterfactual simulations: potential to improve outcomes
  ▶ Optimal: arbitrary length cycles
  ▶ Two and Three-cycle swap

Pareto improvement

Recipients

Donors
Counterfactual Analysis

LKDPI distribution

- Optimal with constraint
- Two&Three-cycle Swap
- Original

Frequency

LKDPi

-20 -10 0 10 20 30 40 50 60 70 80 90 100
Counterfactual Analysis

LKDPI distribution

- Blue: Optimal with constraint
- Red: Two&Three-cycle Swap
- Yellow: Original

Frequency vs. LKDPI
Counterfactual Analysis

LKDPI distribution

- Optimal with constraint
- Two&Three-cycle Swap
- Original

Frequency

LKDPPI

Cadaveric

Better

9
From LKDPI to Graft Survival

• Expected graft survival: estimated as a function of LKDPI: \( 14.78 \exp(-0.01239 \text{ LKDPI}) \)
Including Compatible Pairs in Kidney Exchange

- Increase in the number of matches for **incompatible** pairs (quantity)

- Increase in the expected graft survival for **compatible** pairs (quality)
LKDPI Simulator
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• Basic simulator model:
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  ▶ Compatibility based on the simulator from Saidman et al. (2006)
LKDPI Simulator

• To analyze the effects of policy changes, we need a faithful simulation of the real process

• Basic simulator model:
  ▶ Compatibility based on the simulator from Saidman et al. (2006)

  ▶ Generate LKDPI-related characteristics to measure expected graft survival
Dynamic Matching
Dynamic Matching

✧ Compatible pairs may not be willing to wait any longer than necessary

▷ Also debate in the literature about the value of patience regardless (Akbarpour, S. Li, and Oveis Gharan, 2017; Ashlagi et al., 2017; Z. Li et al., 2015, 2018)
Dynamic Matching

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- New model: Hybrid Static-Dynamic Matching Model
  - A pool of patient incompatible pairs
  - Impatient compatible pairs
Hybrid Static-Dynamic Matching Model
Hybrid Static-Dynamic Matching Model

Online agent (Compatible pair) $t=2$

Standby agents (Incompatible pool)
Hybrid Static-Dynamic Matching Model

Online agent (Compatible pair)  
$t=3$

Standby agents (Incompatible pool)

10
5

3
6
8
Hybrid Static-Dynamic Matching Model

Online agent (Compatible pair) $t=4$

Standby agents (Incompatible pool)
Hybrid Static-Dynamic Matching Model

Standby agents (Incompatible pool)
Hybrid Static-Dynamic Matching Model
Algorithmic Approach

• Most approaches in dynamic settings based on either greedy or batching mechanisms

• We consider a relaxed Integer Programming formulation:

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\begin{align*}
\text{max} & \quad \sum_{n=1}^{N} \sum_{i=0}^{I} w_{n,i} x_{n,i} \\
\text{s.t.} & \quad \sum_{i=0}^{I} x_{n,i} \leq 1, \forall n \in [T] \\
& \quad \sum_{n=1}^{N} x_{n,i} + \sum_{j=1}^{I} x_{T+i,j} \leq 1, \forall i \in [I] \\
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Weights (match quality)

Match variables

Online agents (compatible pairs)

Each online agent matches either with itself (i=0) or with a standby agent (i>0)

Standby agents (incompatible pairs)
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Each standby agent matches with exactly one other agent
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Weights (match quality)

Match variables

Online agents (compatible pairs)

Easily extend to $k$ cycles and chain

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Each standby agent matches with exactly one other agent

Standby agents (incompatible pairs)

All agents
Dual Formulation and ODASSE

\[
\begin{align*}
\min & \quad \sum_{t=1}^{T} \alpha_t + \sum_{i=0}^{I} \beta_i \\
\text{s.t.} & \quad w_{t,i} - \alpha_t - \beta_i \leq 0, \forall t \in [T], i \in [I]^* \\
& \quad w_{t+j,i} - \beta_j - \beta_i \leq 0, \forall i \in [I], j \in [I] \\
& \quad \alpha_t, \beta_i \geq 0, \forall t \in [T], i \in [I] \\
& \quad \beta_0 = 0
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- \( \alpha_t, \beta_i \) can be interpreted as estimated values (shadow survival estimates) of compatible pairs and incompatible pairs respectively.
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- \(\alpha_t, \beta_i\) can be interpreted as estimated values (shadow survival estimates) of compatible pairs and incompatible pairs respectively.
- Given optimal \(\beta_i^*\) we can derive the online assignment rule \(i^* = \arg\max_i \{w_{t,i} - \beta_i^*\}\) (Online Dual Assignment Using Shadow Survival Estimates).
Estimating $\beta_i^*$
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- Training data generated by simulator
Estimating $\beta^*_i$

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  - True value comes by solving oracle version of relaxed integer programming problem
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Estimating $\beta_i^*$

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  - Demographic information of an incompatible pair
Estimating $\beta_i^*$

- **Training data generated by simulator**
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- **Train machine learning model on**
  - Demographic information of an incompatible pair
  - Initial graph state of incompatible pairs
Estimating $\beta_i^*$

- **Training data generated by simulator**
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- **Train machine learners**
  - Demographic information of an incompatible pair
  - Initial graph state of incompatible pairs

![Graph showing predicted vs. true $\beta$ values]

- Scatter plot of predicted $\beta$ vs. true $\beta$
Results

- Increase in the number of matches for **incompatible** pairs (quantity)
- Increase in the expected graft survival for **compatible** pairs (quality)
Results: Potential Social Impact

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>OAES</th>
<th>ODASSE</th>
<th>Oracle</th>
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<tbody>
<tr>
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<td>54.4%</td>
<td>74.6%</td>
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OAES (Online allocation via exhaustive search) solves an IP each time but only performs the match recommended for the online/impatient agent.
Results: Potential Social Impact

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Results: Fairness (O types)

- Proportion Matched

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<th>OAES</th>
</tr>
</thead>
<tbody>
<tr>
<td>incompatible pairs</td>
<td>0.544</td>
<td>0.75</td>
</tr>
<tr>
<td>with recipients with type O</td>
<td>0.32</td>
<td>0.62</td>
</tr>
</tbody>
</table>
Results: Algorithms

Total Expected Graft Survival by Algorithm

Baseline (Compatible and Incompatible Separate)
OAES
ODASSE (ML estimates of $\beta$)
ODASSE (Simulated estimates of $\beta$)
Oracle (ODASSE with perfect $\beta$)
Conclusion

• A framework for analyzing match quality in models of kidney exchange

• Estimate the benefits of including compatible pairs in kidney exchange for both compatible pairs and incompatible pairs
  ✤ A new hybrid static-dynamic matching model.
  ✤ Online primal-dual + learning algorithm

• Practical directions
  ✤ Embed with the surgical team for weekly intake meetings
  ✤ Track waiting times and qualities
  ✤ Implement weighted allocation mechanism in a single center