Topology-Constrained Surface Reconstruction From Cross Sections

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## Motivation

- Medical imaging

| CT scanner | A stack of CT scans (2D) of a human brain | A medical volume (3D) of a developing chicken heart |
• In medical imaging, segmentation is **important** but **challenging**.

3D volume of a developing chicken heart
In medical imaging, segmentation is **important** but **challenging**.

Cross-sections of a developing chicken heart
In medical imaging, segmentation is important but challenging.

Drawing contours on selected cross-section
Segmentation

Contour drawing (manual by doctors)

Reconstruction (automatic)
Segmentation

Contour drawing (manual by doctors)

Reconstruction (automatic)
Problem Definition

- Given a set of contours, create a surface that interpolates the contours.
  - The fewer contours it requires, the better.
  - The nicer the surface, the better.
What’s a “nice” surface?

- **Interpolating**
  - Surface passes through contours
What’s a “nice” surface?

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- Geometrically correct
  - No hole, no intersection
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- **Topologically correct**
  - Desired genus and # of components
What’s a “nice” surface?

- Interpolating
  - Surface passes through contours

- Geometrically correct
  - No hole, no intersection

- **Topologically correct**
  - Desired genus and # of components

- genus = 0
- genus = 1
- genus = 2
What’s a “nice” surface?

- Interpolating
  - Surface passes through contours

- Geometrically correct
  - No hole, no intersection

- Topologically correct
  - Desired genus and # of components

What’s a “nice” surface?

- $c=1$, $g=0$
- $c=2$, $g=0$
- $c=1$, $g=1$
- $c=1$, $g=2$
Previous work

- Contour based surface reconstruction
  - Parallel cross-sections
    [Keppel 75] [Fuchs 77] [Meyers 92]
  - Non-parallel cross-sections
    [Boissonnat 88] [Herman 92] [Bajaj 96] [Oliva 96]
    [Cheng 99] [Turk 99] [Barequet 96, 04, 07, 09] [Ju 05]
    [Boissonnat 07] [Liu 08] [Bermano 11] [Zhou 14]

- Interpolating
- Geometrically correct
Previous work

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- Interpolating
- Geometrically correct
- No topology control
Previous work

- Topology repairing as post process
  [Wood 04] [Zhou 07] [Attene 13]

  ![Sad face] Oblivious of the original input data. May not interpolate contours.

- Reconstruction with topology control (not from contours)
  [Sharf 07] [Bazin 07] [Zeng 08] [Yin 14] [Ijiri 14]

  ![Sad face] Requires human interaction or initial templates.
First contour-based reconstruction method with topology control

• Creates a single connected surface with user-specified genus
• Requires no user interaction or templates
Algorithm
Algorithm

Input Contours (Genus = 1)

Output Surface
Algorithm: 2D illustration

Input Contours (Genus = 1) → Output Surface
Algorithm: 2D illustration

Cross sections

(parallel or non-parallel)

Volume

Contours
Algorithm: 2D illustration

Bounding box

(parallel or non-parallel)

Cross sections

Contours
Algorithm: 2D illustration

(independent)

Cells
Algorithm: 2D illustration
2-step approach: explore and select
2-step approach: explore and select

1. **Explore**
   - Explore topologies in each independent cell

2. **Select**
   - Select one topology per cell to form the best surface
Step 1: explore

(If limited to simple topology)

one topology within a cell

= 

one grouping of the contours
Step 1: explore
Step 1: explore
Step 1: explore
Step 1: explore | difference

- **Ours**: Explore and keep **multiple** topologies within one cell.
- **Previous methods**: Generate **one** single topology per cell.

VS.

[Ours]

[Liu 08]
[Bermano 11]
• Huge search space
  • # of possible connections is a bell number
    \[ B(4)=15; \; B(6)=203; \; B(10)=115,975 \]

• Non-trivial surfacing problem
  • No self-intersection
  • Simple topology (no extra handle, tunnel)

• Score the naturalness of the surfaces

\[ \begin{array}{l}
\vdash \; > \\
\vdash \end{array} \]
Huge search space

Non-trivial surfacing problem

Score the naturalness of the surfaces

Iso-surface of an implicit function

Define with the implicit function.

Much smaller search space.

Always geometrically correct.
• What implicit function to choose?
  • Satisfy boundary conditions
  • Continuous inside the cell and bounded by $[0,1]$.
  • Good indication of the likelihood of a point being inside/outside.

Iso-surface of an implicit function
Step 1: explore random walk

The probability of point $i$ being labeled with 1 is determined by its neighbors $N(i)$ and the weight $w_{ij}$ between point $i$ and $j$. The discrete harmonic function is given by:

$$x_i - \frac{\sum_{j \in N(i)} w_{ij} x_j}{\sum_{j \in N(i)} w_{ij}} = 0$$

where $x_i$ is the probability of point $i$ being labeled with 1, $N(i)$ are the neighbors of point $i$, and $w_{ij}$ is the weight between point $i$ and $j$.
Step 1: explore | random walk

\[ x_i - \frac{\sum_{j \in N(i)} w_{ij} x_j}{\sum_{j \in N(i)} w_{ij}} = 0 \]

\[ w_{ij} = \exp(-\beta(I_i - I_j)^2) \]

- \( x_i \): the probability of point \( i \) being labeled with 1
- \( N(i) \): neighbors of point \( i \)
- \( w_{ij} \): weight between point \( i \) and \( j \)
- \( I_i \): intensity of point \( i \) in input volume
Step 1: explore | iso-surface
Step 1: explore | iso-surface
Step 1: explore | iso-surface
Step 1: explore \mid score

Score: joint likelihood of all the points having the inside or outside label.

\[ h(T) = \int_{\Omega} h_T(x) \, dx \]

\[ h_T(x) = \begin{cases} 
\log(f(x)) & \text{inside} \\
\log(1 - f(x)) & \text{outside} 
\end{cases} \]
Step 1: explore | score

score = -5383
score = -3810
score = -5123
score = -13510
Step 1: explore | score

score = -3810
score = -5123
score = -5383
score = -13510
Step 1: explore

Per-cell Topology

h=-3810
h=-5123
h=-1019
h=-2812
h=-1201
h=-5383
h=-1019
h=-2812
h=-1201
Step 2: select

Per-cell Topology

h = -1201

h = -1019

h = -1201

h = -3810

h = -5123

h = -5383

h = -1019

h = -2812

h = -1019

h = -2812
**Step 2: select**

Select one topology per cell, such that:

- The merged surface is one single component and has desired genus
- The summation of the scores of the selected topologies is maximized
Step 2: select

Algorithm:
Bottom-up Dynamic Programming
Step 2: select

**Algorithm:**
Bottom-up Dynamic Programming

- Merge *cells* into *regions*, one cell at a time
Step 2: select

Algorithm:
Bottom-up Dynamic Programming

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Step 2: select

Algorithm:
Bottom-up Dynamic Programming

• Merge *cells* into *regions*, one cell at a time
Algorithm: Bottom-up Dynamic Programming

- Merge *cells* into *regions*, one cell at a time
- Merge *per-cell topologies* to *per-region topologies*
Algorithm:
Bottom-up Dynamic Programing

- Merge cells into regions, one cell at a time
- Merge per-cell topologies to per-region topologies
  - keep the highest-score combination of per-cell topologies

Step 2: select

comp = 2
genus = 0

h = -6030

h = -7103
Algorithm:
Bottom-up Dynamic Programming

- Merge cells into regions, one cell at a time
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Algorithm:
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- Merge cells into regions, one cell at a time
- Merge per-cell topologies to per-region topologies
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Step 2: select
Results
<table>
<thead>
<tr>
<th></th>
<th># of slices = 10</th>
<th>[Liu et al. 2008]</th>
<th>[Bermano et al. 2011]</th>
<th>Ours</th>
</tr>
</thead>
</table>

Requires fewer slices
<table>
<thead>
<tr>
<th>Requires fewer slices</th>
</tr>
</thead>
<tbody>
<tr>
<td># of slices = 7</td>
</tr>
<tr>
<td>[Liu et al. 2008]</td>
</tr>
<tr>
<td>[Bermano et al. 2011]</td>
</tr>
<tr>
<td>Ours</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>Requires fewer slices</td>
</tr>
</tbody>
</table>

-genus = 1-
<table>
<thead>
<tr>
<th>input slices + 3D volume</th>
<th>[Liu et al. 2008]</th>
<th>[Bermano et al. 2011]</th>
<th>Ours</th>
</tr>
</thead>
</table>

Add additional volume data
Add additional volume data

input slices + 3D volume

[Liu et al. 2008] [Bermano et al. 2011] Ours

genus = 1
More results

Input slices  [Liu et al. 2008]  [Bermano et al. 2011]  Ours w/o volume

genus = 1
time: 23.16s
More results

Input slices | [Liu et al. 2008] | [Bermano et al. 2011] | Ours w/o volume

genus = 2

Time: 3.5s
Input slices

[Liu et al. 2008]

[Bermano et al. 2011]

Ours w/o volume

genus = 1

time: 56.2s
Conclusion

• Contour based reconstruction method with topology control
  • Requires fewer contours while guaranteeing topology correctness
  • 2-step algorithm:
    (1) Local exploration
    (2) Global selection

• Future work
  • Explore larger space of surface topology
  • Reconstruct from other data with topology control
Thank You!

Q&A

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http://www.cse.wustl.edu/~zoum
(code & data)