COMPUTER-AIDED DESIGN

Computer-Aided Design 33 (2001) 903–912

www.elsevier.com/locate/cad

Modifying the shape of NURBS surfaces with geometric constraints

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Received 18 December 1999; revised 1 August 2000; accepted 17 August 2000

Abstract

NURBS surfaces are among the most commonly used parametric surfaces in CAGD and Computer Graphics. This paper investigates shape modification of NURBS surfaces with geometric constraints, such as point, normal vector, and curve constraints. Two new methods are presented by constrained optimization and energy minimization. The former is based on minimizing changes in control net of surfaces, whereas the latter is based on strain energy minimization. By these two methods, we change control points and weights of an original surface, such that the modified surface satisfies the given constraints. Comparison results and practical examples are also given.

Keywords: Shape modification; NURBS; Energy minimization; Constrained optimization; Highlight line

1. Introduction

NURBS is one of the most popular and successful methods for designing complex surfaces in Computer Aided Geometric Design (CAGD) and Computer Graphics [6,14]. After creating NURBS surfaces, we often need to modify them to satisfy the user's design requirements. By definition of NURBS surfaces, there are three ways to modify the shape:

- by changing knot vector;
- by moving control points;
- by changing the weights.

Many efforts have been made towards developing more convenient methods for shape modification of NURBS curves. Piegl [12] proposed two methods to vary the shape of NURBS curves and surfaces: control-point-based modification and weight-based modification. Fowler and Bartels [7] presented a method to let users interactively specify geometric properties that depend on a curve's values and derivatives at selected points. Au and Yuen [1] and Sánchez-Reyes [15] introduced approaches that modify the shape of NURBS curves by altering the weights and location of the control points simultaneously. Ishida [9] proposed a method that enables arbitrary and direct modification of curves by constructing a displacement function. Zheng et al. [22] presented a new approach for directly manipulating the shape by modification in control points and knot refinements. Juhász [10] provides a weight-based shape modification method with point and tangent constraints for plane NURBS curves.

As for shape modification of surfaces, besides Piegl's method [13], Celniker and Wech [4] investigated a method for constrained deformation of B-spline surfaces by using linear constraints and global energy function minimizing. Welch and Watkin [20] also considered linear constraints for deformable B-splines, but different energy functions were used. Kimura et al. [11] considered the deformation of a given surface that smoothly connects to previously designed surfaces. Guillet and Léon [8] described an approach for deformation of multi-patch tensor based free-form surfaces, and the deformation generated is controlled by global geometric constrains. Singh and Fiume [16] presented an effective geometric deformation technique: wires for shape modification of surfaces and objects.

Physically based modeling approaches such as the finite element method and physical-based NURBS are also powerful methods for shape modification [3,17]. Users interact with the model by exerting virtual forces to which the system responds. However, there is a point of concern: high computational cost is involved in those non-purely geometric methods. Therefore physically based modeling

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0010-4485/01/$ - see front matter
PII: S0010-4485(00)00115-9

approaches have not found wide applications in current CAD systems.

For a given NURBS surface and a given target point, a problem is how to modify the control points or corresponding weights such that the modified surface can pass through the target point. Piegl gave an efficient way to achieve this by moving one control point or changing one weight, and presented explicit formulae to compute the new control point or weight. Due to its efficiency, Piegl's method has been widely used in commercial CAD systems. The objective of this paper is to improve Piegl's purely geometric methods for modifying shape of NURBS more naturally.

The paper is organized as follows: the problem statement is given in Section 2. Section 3 presents the constrained optimization method by minimizing changes of control net or weights of NURBS surfaces in Least Square sense. Section 4 presents the constrained optimization method by minimizing changes of strain energy of surfaces. Comparison results and practical examples are given in Section 5. Remarks on shape modification of NURBS curves are noted in Section 6.

2. Problems statement

A NURBS surface with control points P_{ij} $(0 \le i \le n, 0 \le j \le m)$ can be defined as

$$P(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} \omega_{ij} P_{ij} N_{ik}(u) N_{jl}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m} \omega_{ij} N_{ik}(u) N_{jl}(v)}, \tag{1}$$

$u_{k-1} \le u \le u_{n+1}, \ \ v_{l-1} \le v \le v_{m+1},$

where ω_{ij} are corresponding weights of P_{ij}, $N_{i,k}(u)$ and $N_{j,l}(u)$ are the normalized B-spline base functions of order k and l, respectively, defined over knot vector $U = \{u_0, u_1, \dots, u_n, \dots, u_{n+k}\}$ and $V = \{v_0, v_1, \dots, v_m, \dots, v_{m+k}\}$. Eq. (1) can be rewritten as

$$P(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} P_{ij} R_{ij}(u,v), \tag{2}$$

$u_{k-1} \le u \le u_{n+1}, \ \ v_{l-1} \le v \le v_{m+1}$

where

$$R_{ij}(u,v) = \omega_{ij} N_{ik}(u) N_{jl}(v) \Bigg/ \left(\sum_{i=0}^{n}\sum_{j=0}^{m} \omega_{ij} N_{ik}(u) N_{jl}(v) \right).$$

Piegl's classical problem is given as follows: for a start point S on a NURBS surface, how to modify the surface such that the point S passes through a target point T. In geometric modeling systems, we usually pick up a point in a surface, and then drag the surface to a target point by mouse.

This paper considers shape modification of NURBS not only with single point constraint as mentioned above, but also with normal vector, multi-point, and curve constraints.

3. Shape modification by minimizing variation of control net

In this section, we consider shape modification of NURBS surface with different constraints by using discrete Euclidean norm. We compute the modified surface by minimizing the changes of control net in Least-square sense. The biggest benefit of this technique is that we can obtain explicit formula for computing new control points in many cases. This method can be extended to weighted-based approach, i.e. adjust weights to satisfy the geometric constraint.

3.1. Least square solutions for point and normal vector constraints

Piegl gave an efficient way by moving one control point or changing one weight for the single point constraint problem, and presented explicit formulae to compute the new control point and weight. However, since only one control point or weight is modified, shape modification of the surface seems to be unnatural (as can be seen later in the examples by highlight line visualization). The main idea of the paper is to alter more than one control point, so that the modification of the shape is distributed over multiple control points. This section considers a more general problem: for $k \times l$ order NURBS surface $P(u,v)$, S_l $(l = 0, 1, \dots r)$ are start points with parameters (u_l, v_l), T_l $(l = 0, 1, \dots r)$ are target points, N_l, $l = 0, 1, \dots r$ are desired normal vectors, how can the control points be adjusted, such that the modified surface $\hat{P}(u,v)$ passes through those target points, and take N_l as normal vectors at points T_l.

Suppose control points P_{ij} $(i_1 \le i \le i_2, j_1 \le j \le j_2)$ are to be changed. We choose perturbation $\epsilon_{ij} = \lfloor \epsilon_{ij}^x, \epsilon_{ij}^y, \epsilon_{ij}^z \rfloor (i_1 \le i \le i_2, j_1 \le j \le j_2)$ for those control points, such that the modified surface

$$\hat{P}(u,v) = P(u,v) + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2} \epsilon_{ij} R_{ij}(u,v), \tag{3}$$

$u_{k-1} \le u \le u_{n+1}, \ \ v_{l-1} \le v \le v_{m+1}$

not only passes through the target points T_l, but also take N_l as normal vectors at points T_l, i.e. $\hat{P}(u,v)$ satisfies the following equations:

$$T_l = \hat{P}(u_l, v_l) = S_l + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2} \epsilon_{ij} R_{ij}(u_l, v_l), \qquad l = 0, 1, \dots, r \tag{4}$$

$$\begin{cases} \frac{\partial}{\partial u}\hat{P}(u_l,v_l)\cdot N_l = \left(\frac{\partial}{\partial u}P(u_l,v_l) + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\epsilon_{ij}R_{ij}^u(u_l,v_l)\right)\cdot N_l = 0 \\ \frac{\partial}{\partial v}\hat{P}(u_l,v_l)\cdot N_l = \left(\frac{\partial}{\partial v}P(u_l,v_l) + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\epsilon_{ij}R_{ij}^v(u_l,v_l)\right)\cdot N_l = 0 \end{cases} \quad l = 1,2,\dots,r \tag{5}$$

We determine ϵ_{ij} by the constrained optimization method. The optimization objective function is

$$\sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\|\epsilon_{ij}\|^2 = \mathrm{Min} \tag{6}$$

and the Lagrange function is defined as

$$\begin{aligned} L = {} & \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\|\epsilon_{ij}\|^2 + \sum_{l=0}^{r}\lambda_l\left(T_l - S_l - \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\epsilon_{ij}R_{ij}(u_l,v_l)\right) \\ & + \sum_{l=0}^{r}\delta_l\left(\left(\frac{\partial}{\partial u}P(u_l,v_l) + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\epsilon_{ij}R_{ij}^u(u_l,v_l)\right)\cdot N_l\right) \\ & + \sum_{l=0}^{r}\gamma_l\left(\left(\frac{\partial}{\partial v}P(u_l,v_l) + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\epsilon_{ij}R_{ij}^v(u_l,v_l)\right)\cdot N_l\right) \end{aligned} \tag{7}$$

where $\lambda_l = [\lambda_l^x, \lambda_l^y, \lambda_l^z]^{\mathrm{T}}$, δ_l, $\gamma_l (l = 0,1,\dots,r)$ are Langrange multipliers and $\|\cdot\|$ is Euclidean norm.

Let $\frac{\partial}{\partial\lambda_l^x}(L)$, $\frac{\partial}{\partial\lambda_l^y}(L)$, $\frac{\partial}{\partial\lambda_l^z}(L)$, $\frac{\partial}{\partial\delta_l}(L)$, $\frac{\partial}{\partial\gamma_l}(L)$ be zero for $l = 0,1,\dots,r$, and $\frac{\partial}{\partial\epsilon_{ij}^x}(L)$, $\frac{\partial}{\partial\epsilon_{ij}^y}(L)$, $\frac{\partial}{\partial\epsilon_{ij}^z}(L)$ be zero for $i_1 \le i \le i_2$, $j_1 \le j \le j_2$, and write the derived formulae in vector form, we have the following system:

$$\begin{cases} T_l = S_l + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\epsilon_{ij}R_{ij}(u_l,v_l), & l = 0,1,\dots,r, \\ \left(\frac{\partial}{\partial u}P(u_l,v_l) + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\epsilon_{ij}R_{ij}^u(u_l,v_l)\right)\cdot N_l = 0, & l = 0,1,\dots,r, \\ \left(\frac{\partial}{\partial v}P(u_l,v_l) + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\epsilon_{ij}R_{ij}^v(u_l,v_l)\right)\cdot N_l = 0, & l = 0,1,\dots,r, \\ 2\epsilon_{ij} - \sum_{l=0}^{r}\left(\lambda_l R_{ij}(u_l,v_l) - \delta_l R_{ij}^u(u_l,v_l)\cdot N_l - \gamma_l R_{ij}^v(u_l,v_l)\cdot N_l\right) = 0, & i_1 \le i \le i_2, j_1 \le j \le j_2 \end{cases} \tag{8}$$

By solving the above equation system, the constrained optimization solution can be obtained.

If the geometric constraint is only a single point, i.e. Piegl's classical problem, the Eq. (8) should be

$$\begin{cases} T = S + \sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\epsilon_{ij}R_{ij}(u_s,v_s) \\ \epsilon_{ij} = \frac{\lambda}{2}R_{ij}(u_s,v_s), \qquad i = 0,1,\dots,n;\ j = 0,1,\dots,m \end{cases} \tag{9}$$

which yields the following explicit solution

$$\epsilon_{ij} = \frac{R_{ij}(u_s,v_s)}{\sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}R_{ij}^2(u_s,v_s)}(T - S) \tag{10}$$

Furthermore, If only one control point is modified, from Eq. (10), we have

$$\epsilon_{ij} = \frac{T - S}{R_{ij}(u_s,v_s)}. \tag{11}$$

This is the equation (30) in Piegl's paper [13].

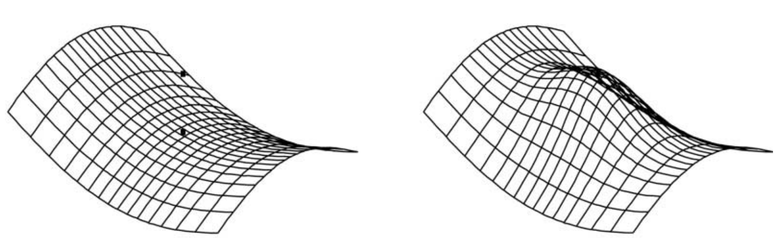
The follows are some examples. Fig. 1 is an example with single point constraint, Fig. 2 applies a normal vector constraint to the target point, and Fig. 3 is an example for multi-point constraints. In the following figures, (a) is the original surface, (b) is the modified surface.

3.2. Explicit solution for isoparametric curve constraint

For a $k \times l$ order NURBS surface $P(u,v)$, how can the control points be adjusted such that a selected isoparametric curve matches a given objective curve? We assume that the original isoparametric curve and the objective curve possess the same knot vector and are of the same order. Suppose the isoparametric curve $L_s(u) = P(u,v_s)$, and the objective curve is defined as

$$L_t(u) = \frac{\sum_{i=0}^{n}\eta_i L_i N_{i,k}(u)}{\sum_{i=0}^{n}\eta_i N_{i,k}(u)}, \qquad u_{k-1} \le u \le u_{n+1}, \tag{12}$$

The control points $P_{ij}, 0 \le i \le n, 0 \le j \le m$ are to be changed. We choose perturbation $\epsilon_{ij} = [\epsilon_{ij}^x, \epsilon_{ij}^y, \epsilon_{ij}^z], 0 \le i \le n, 0 \le j \le m$ for those control points, such that the isoparametric curve with parameter $v = v_s$ in the



(a) (b)

Fig. 1. Shape modification of NURBS surface with point constraint.

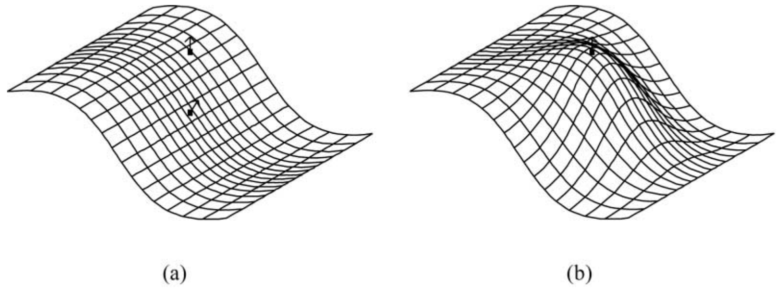


Fig. 2. Shape modification of NURBS surface with single point and normal vector constraints.

modified surface

$$\hat{P}(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{ij}\hat{P}_{ij}N_{ik}(u)N_{jl}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{ij}N_{ik}(u)N_{jl}(v)}$$

$$= \frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{ij}(P_{ij}+\epsilon_{ij})N_{ik}(u)N_{jl}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{ij}N_{ik}(u)N_{jl}(v)},$$

$$u_{k-1} \le u \le u_{n+1}$$

$$v_{l-1} \le v \le v_{m+1}$$

is the object curve $L_t(u)$. Note that

$$\hat{L}_s(u) = \hat{P}(u,v_s) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{ij}\hat{P}_{ij}N_{ik}(u)N_{jl}(v_s)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{ij}N_{ik}(u)N_{jl}(v_s)}$$

$$= \frac{\sum_{i=0}^{n}\left(\frac{\sum_{j=0}^{m}\omega_{ij}\hat{P}_{ij}N_{jl}(v_s)}{\sum_{j=0}^{m}\omega_{ij}N_{jl}(v_s)}\right)\left(\sum_{j=0}^{m}\omega_{ij}N_{jl}(v_s)\right)N_{ik}(u)}{\sum_{i=0}^{n}\left(\sum_{j=0}^{m}\omega_{ij}N_{jl}(v_s)\right)N_{ik}(u)},$$

is a NURBS curve with control points $\sum_{j=0}^{m}\omega_{ij}\hat{P}_{ij}N_{jl}(v_s)/\sum_{j=0}^{m}\omega_{ij}N_{jl}(v_s)$, and weights $\sum_{j=0}^{m}\omega_{ij}N_{jl}(v_s)$. Since it is the same curve as $L_t(u)$, we have

$$\begin{cases}\sum_{j=0}^{m}\omega_{ij}\hat{P}_{ij}N_{jl}(v_s) = \kappa\eta_i L_i \\ \sum_{j=0}^{m}\omega_{ij}N_{jl}(v_s) = \kappa\eta_i\end{cases},$$

where κ is a constant of proportionality. Then we have the following constraint equation:

$$\sum_{j=0}^{m}\omega_{ij}\hat{P}_{ij}N_{jl}(v_s) = L_i\sum_{j=0}^{m}\omega_{ij}N_{jl}(v_s), \qquad i = 0,1,\ldots n. \tag{13}$$

We compute ϵ_{ij} by minimizing $\sum_{i=0}^{n}\sum_{j=0}^{m}\|\epsilon_{ij}\|^2$. The Lagrange function can be defined as

$$L = \sum_{i=0}^{n}\sum_{j=0}^{m}\|\epsilon_{ij}\|^2 + \sum_{i=0}^{n}\lambda_i\left(\sum_{j=0}^{m}\omega_{ij}(P_{ij}+\epsilon_{ij})N_{jl}(v_s) - L_i\sum_{j=0}^{m}\omega_{ij}N_{jl}(v_s)\right) \tag{14}$$

So we have the following equations:

$$\begin{cases}\sum_{j=0}^{m}\omega_{ij}(P_{ij}+\epsilon_{ij})N_{jl}(v_s) - L_i\sum_{j=0}^{m}\omega_{ij}N_{jl}(v_s) = 0, & 0 \le i \le n, \\ 2\epsilon_{ij} + \lambda_i\omega_{ij}N_{jl}(v_s) = 0, & 0 \le \mathrm{i} \le n, 0 \le j \le m,\end{cases} \tag{15}$$

and the explicit solution for the curve constraint can be obtained as follows:

$$\epsilon_{ij} = \frac{\omega_{ij}N_{jl}(v_s)}{\sum_{j=0}^{m}\omega_{ij}^2N_{jl}^2(v_s)}\sum_{j=0}^{m}\omega_{ij}(P_{ij}-L_i)N_{jl}(v_s) \tag{16}$$

Fig. 4 is an example of iso-curve constraint, where (a) is the original surface and (b) is the modified surface. The objective curve in (a) is rendered as dotted line.

In this section, we just consider a special case for isoparametric curve. For a general case, Celniker and Welch [4] presented an approach. But in such a situation, the curve in NURBS defined by $P(u(t), v(t))$ is usually a curve with a higher degree. It is more efficient by using iso-curve constraints for its explicit solution. Some applications will be given in Section 5.

4. Shape modification by energy minimization

In this section, we consider shape modification of NURBS surface with different constraints by using energy minimization. The thin plate energy of a surface $P(u,v)$ is

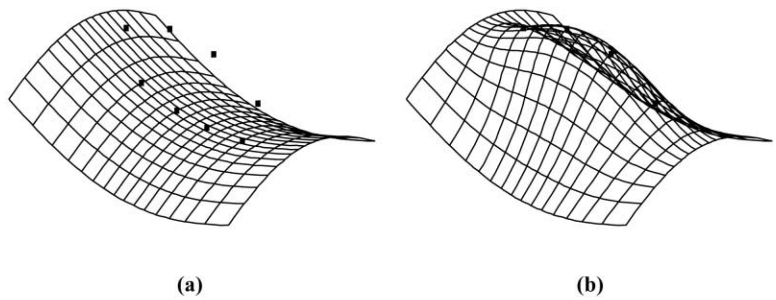


Fig. 3. Shape modification of NURBS surface with multi-point constraints.

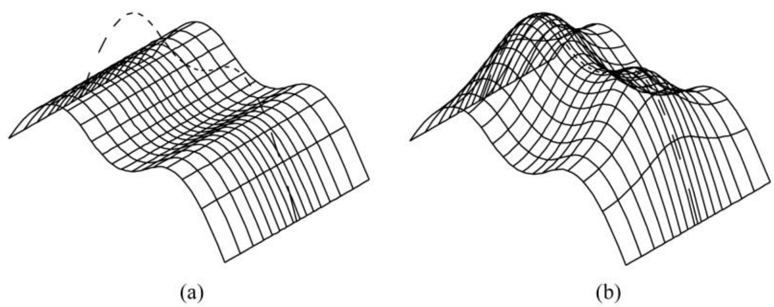


Fig. 4. Shape modification of NURBS surface with iso-curve constraint.

usually defined as

$$E(P) = \int\int (P_{uu}^2 + 2P_{uv}^2 + P_{vv}^2)\,\mathrm{d}u\,\mathrm{d}v, \tag{17}$$

The energy of a parametric surface implies its global properties in certain sense, so that it is often used in surface fitting and fairing for smooth and natural shape [5,18,19]. Here we would like to change control points of NURBS surfaces, such that the thin plate energy of error surface is minimized. The examples through highlight line visualization in the next section show that this approach can produce better modified surfaces.

Suppose control points P_{ij}, $0 \le i \le n, 0 \le j \le m$ are to be changed. We choose perturbation $\epsilon_{ij} = \lfloor \epsilon_{ij}^x, \epsilon_{ij}^y, \epsilon_{ij}^z \rfloor$, $0 \le i \le n, 0 \le j \le m$ for those control points, such that the modified surface $\hat{P}(u,v)$ satisfies some geometric constraints.

We would like to determine ϵ_{ij} by constrained optimization method, such that

$$E(\hat{P} - P) = \int\int \Big((\hat{P}_{uu} - P_{uu})^2 + 2(\hat{P}_{uv} - P_{uv})^2 + (\hat{P}_{vv} - P_{vv})^2 \Big)\,\mathrm{d}u\,\mathrm{d}v = \mathrm{Min}. \tag{18}$$

Note that

$$\int\int (\hat{P}_{uu} - P_{uu})^2\,\mathrm{d}u\,\mathrm{d}v = \int\int \Big(\sum_{i,j=0}^{n,m} \epsilon_{ij} R_{ij}^{uu}(u,v)\Big)^2\,\mathrm{d}u\,\mathrm{d}v, \tag{19}$$

where $R_{ij}^{uu}(u,v) = (\partial^2/\partial u^2)(R_{ij}(u,v))$. Define

$$L_{ijgh} = \int\int R_{ij}^{uu}(u,v)\cdot R_{gh}^{uu}(u,v)\,\mathrm{d}u\,\mathrm{d}v,$$

$$M_{ijgh} = \int\int R_{ij}^{uv}(u,v)\cdot R_{gh}^{uv}(u,v)\,\mathrm{d}u\,\mathrm{d}v,$$

$$N_{ijgh} = \int\int R_{ij}^{vv}(u,v)\cdot R_{gh}^{uu}(u,v)\,\mathrm{d}u\,\mathrm{d}v,$$

From Eq. (19), we have

$$\int\int (\hat{P}_{uu} - P_{uu})^2\,\mathrm{d}u\,\mathrm{d}v = \sum_{ij=0}^{n,m} \sum_{gh=0}^{n,m} (\epsilon_{ij}, \epsilon_{gh})\cdot L_{ijgh}$$

Similarly, we have

$$\int\int (\hat{P}_{uv} - P_{uv})^2\,\mathrm{d}u\,\mathrm{d}v = \sum_{ij=0}^{n,m} \sum_{gh=0}^{n,m} (\epsilon_{ij}, \epsilon_{gh})\cdot M_{ijgh}$$

$$\int\int (\hat{P}_{vv} - P_{vv})^2\,\mathrm{d}u\,\mathrm{d}v = \sum_{ij=0}^{n,m} \sum_{gh=0}^{n,m} (\epsilon_{ij}, \epsilon_{gh})\cdot N_{ijgh}$$

So the Lagrange function for point and normal vector

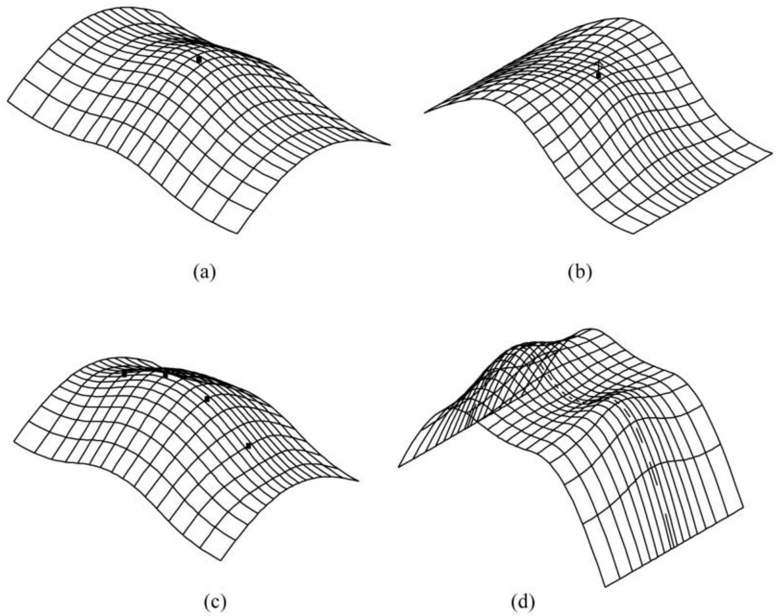


Fig. 5. Shape modification by energy minimization.

constraints (see Eqs. (4) and (5)) can be defined as

$$L = \sum_{i,j=0}^{n,m} \sum_{g,h=0}^{n,m} (\epsilon_{ij}, \epsilon_{gh}) \cdot \left(L_{ijgh} + 2M_{ijgh} + N_{ijgh} \right) + \sum_{l=0}^{r} \lambda_l \left(T_l - S_l - \sum_{i=i_1}^{i_2} \sum_{j=j_1}^{j_2} \epsilon_{ij} R_{ij}(u_l, v_l) \right) + \sum_{l=0}^{r} \delta_l \left(\left(\frac{\partial}{\partial u} P(u_l, v_l) + \sum_{i=i_1}^{i_2} \sum_{j=j_1}^{j_2} \epsilon_{ij} R_{ij}^{u}(u_l, v_l) \right) \cdot N_l \right) + \sum_{l=0}^{r} \gamma_l \left(\left(\frac{\partial}{\partial v} P(u_l, v_l) + \sum_{i=i_1}^{i_2} \sum_{j=j_1}^{j_2} \epsilon_{ij} R_{ij}^{v}(u_l, v_l) \right) \cdot N_l \right) \quad (20)$$

and for isoparametric curve constraints (see Eq. (13)), we have

$$L = \sum_{i,j=0}^{n,m} \sum_{g,h=0}^{n,m} (\epsilon_{ij}, \epsilon_{gh}) \cdot \left(L_{ijgh} + 2M_{ijgh} + N_{ijgh} \right) + \sum_{i=0}^{n} \lambda_i \left(\sum_{j=0}^{m} \omega_{ij} (P_{ij} + \epsilon_{ij}) N_{jl}(v_s) - L_i \sum_{j=0}^{m} \omega_{ij} N_{jl}(v_s) \right) \quad (21)$$

where $\lambda = [\lambda_1, \lambda_2, \lambda_3]^T$ is the Langrange multiplier and $(\cdot, \cdot)$ is the inter product of vectors. Similarly, Eqs. (20) and (21) yield equation systems by which the energy minimization solution can be obtained.

The following are examples for shape modification by energy minimization. Fig. 5(a) is an example of single point constraint, Fig. 5(b) is an example of single point and normal vector constraint, Fig. 5(c) is an example of multi-point constraints, and Fig. 5(d) is for curve constraint. The original surface has been shown in Fig. 1(a), Fig. 2(a), Fig. 3(a) and Fig. 4(a), respectively.

5. Comparison results and practical examples

5.1. Comparison by highlight line visualization

In the design of free-form surfaces, highlight lines have proved to be an effective tool in assessing the quality of a surface. The highlight line visualization is sensitive to the change of normal directions and is suitable for interactive evaluation of the smoothness of a surface [2,21]. In this section, we give several examples of shape modification by the methods presented in this paper, and compare the effects of these methods by highlight line visualization.

Example 1. Shape modification of NURBS with single point constraint. The original shape of the NURBS surface is shown in Fig. 1(a). Fig. 6(a)–(d) demonstrate the results by highlight line visualization of the original surface and modified surfaces by Piegl's method, the method of minimizing the changes of control net and energy minimization method, respectively.

Example 2. Shape modification of NURBS with signle point and vector constraints. The original shape of the NURBS surface is shown in Fig. 2(a). Fig. 7(a)–(c) demonstrate the results by highlight line visualization of the original surface and modified surfaces by the method of minimizing the changes of control net and energy minimization method, respectively.

Example 3. Shape modification of NURBS with multipoint constraints. The original shape of NURBS surface is

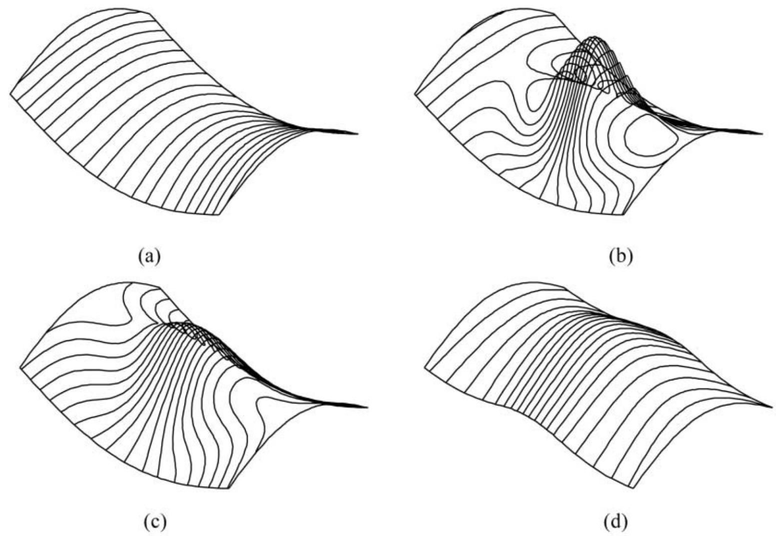


Fig. 6. Highlight line visualization for single point constraint.

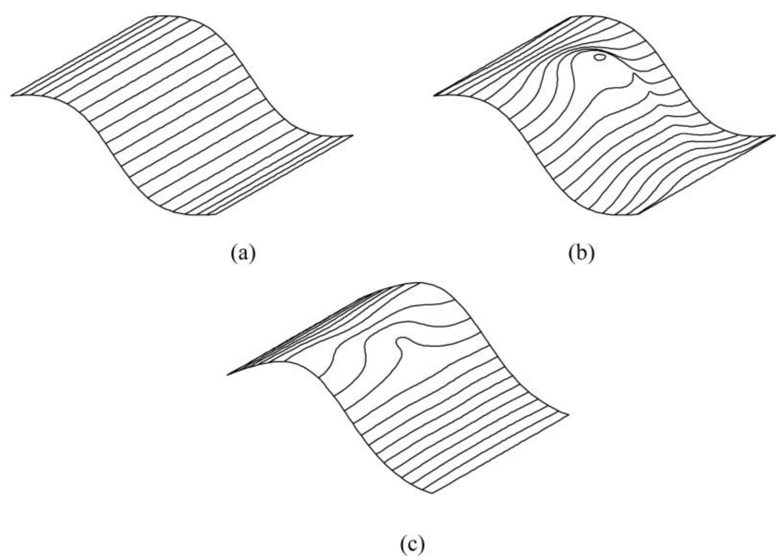


Fig. 7. Highlight line visualization for point and normal vector constraint.

shown in Fig. 3(a). Fig. 8(a)–(c) demonstrate the results by highlight line visualization of the original surface and modified surfaces by the method of minimizing the changes of control net and energy minimization method, respectively.

Example 4. Shape modification of NURBS with a curve constraint. The original shape of NURBS surface is shown in Fig. 4(a). Fig. 9(a)–(c) demonstrate the results by highlight line visualization of the original surface and modified surfaces by the method of minimizing the changes of control net and energy minimization method, respectively.

From these figures by highlight line visualization, it can be seen that the modified shape by energy minimization is better than those by minimizing changes of control net. The latter method can even result in changes in the topological structures as can be seen in some figures using the highlight line visualization.

To further compare the differences between those methods quantitatively, we compute the values of the thin plate energy corresponding to different methods. The results are given in Table 1.

Although shape modification of NURBS by energy minimization gives the most natural results, the method by minimizing changes of control net of the surface can lead to explicit formulae to compute new control points in many cases, and thus achieve real time shape modification. Therefore, where efficiency of computation is of prime importance, the method by minimizing changes of control net of the surface is a more advisable choice.

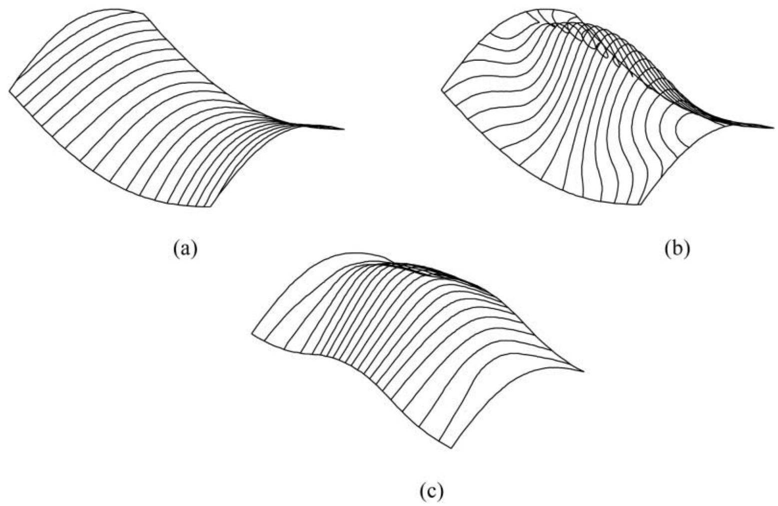


Fig. 8. Highlight line visualization for multi-point constraint.

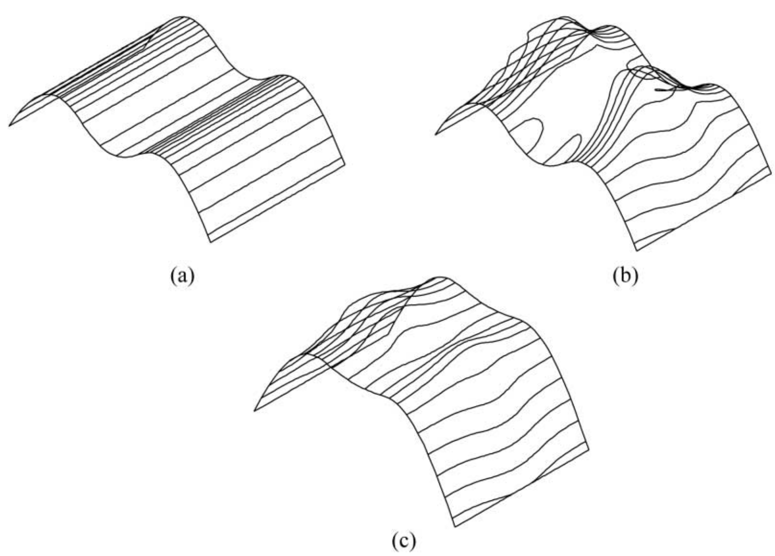


Fig. 9. Highlight line model for curve constraint.

5.2. Practicle example

The following are two practical examples by the method discussed in the paper. For two designed surfaces, we modify the shape by applying iso-curve constraint. Figs. 10 and 11 show original surfaces and modified surfaces by Phong shading.

The above examples show that the proposed methods are efficient for shape modification of NURBS surfaces, and are useful in CAD and Computer Graphics.

6. Remarks

The method presented in the paper can also be used in shape modification of NURBS curves. Similarly, we can modify the shape of a NURBS curves with constraints such as single point, tangent vector and muti-point constraints. A NURBS curve with control points P_i and corresponding weights W_i can be defined as $P(t) = \sum_{i=0}^{n} W_i P_i N_{i,k}(t) / \sum_{i=0}^{n} W_i N_{i,k}(t),\ t_{k-1} \le t \le t_{n+1}$, where $N_{i,k}(t)$ are k order B-spline base function

Table 1

$E(\hat{P} - P)$	Example 1	Example 2	Example 3	Example 4
Piegl	739.732921			
Control net	179.501208	245.514851	394.983501	534.123255
Energy minimization	45.309481	103.160373	200.152197	365.958748

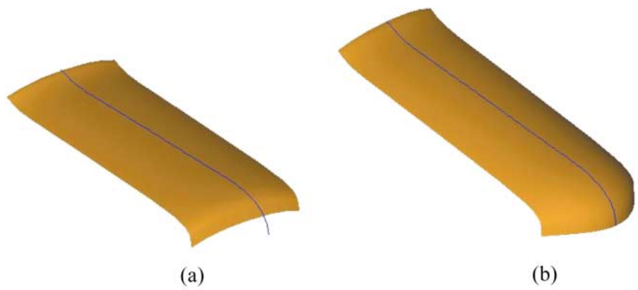


Fig. 10. Shape of the front lid of a car.

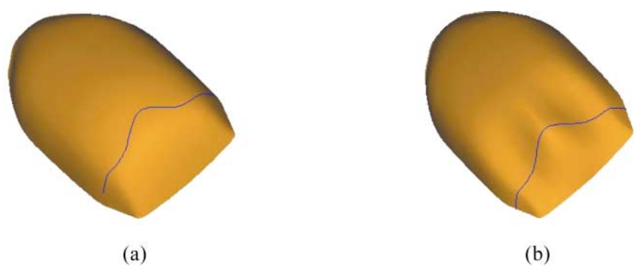


Fig. 11. Shape of top surface of a mouse.

depending on knot vector $\{t_0, t_1, \ldots, t_k, \ldots t_n, t_{n+1,\ldots} t_{n+k}\}$. Similar as Eq. (10), for single point constraint, we can also have the following explicit solution

$$\epsilon_i = \frac{R_{i,k}(t_s)}{\sum_{j=j_1}^{j_2} R_{j,k}^2(t_s)}(T - S), \qquad j_1 \le j \le j_2$$

where $R_{i,k}(t) = W_i N_{i,k}(t) / \sum_{i=0}^{n} W_i N_{i,k}(t)$. If only one control point is modified, from above equation, we have

$$\epsilon = \frac{T - S}{R_{i,k}(t_s)}$$

This is just the equation (28) in Piegl's paper [12]. For other geometric constraints, we can derive solutions by similar methods discussed in Sections 3 and 4. Unlike NURBS surfaces, we should use strain energy instead of thin plate energy, and use curvature profiles to illustrate modification effects of curves instead of highlight line visualization.

Acknowledgements

This work received support from the Natural Science Foundation of China (Project No. 69902004) and Research Grants Council of Hong Kong (Project No. CityU 1049/00E). We especially thank the reviewers for their helpful comments.

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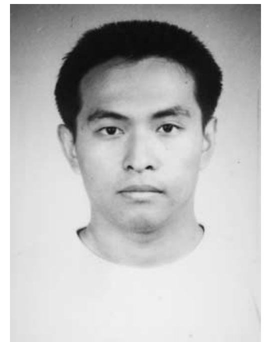

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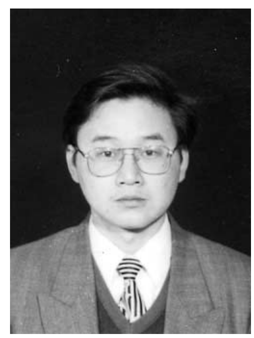

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