CSE546: Computational Geometry
Homework 4: Ham-Sandwich, Duality and line arrangement

Due: before class on Tuesday, Nov 21

**Question 1:** You are given two sets $A$ and $B$ of points in the plane, both of size $n$. Recall that a Ham-Sandwich cut is a line that bisects both point sets, such that there are at least $\lfloor n/2 \rfloor$ points of $A$ and of $B$ on each side of the cut. We define a connection between $A$ and $B$ as a set of $n$ line segments, each connecting a point of $A$ to some point of $B$ (i.e., the segments define a one-to-one correspondence between $A$ and $B$). The connection is disjoint if no two segments intersect.

Assuming you are given an algorithm for computing the Ham-Sandwich cut in $O(n \log n)$ time. Give an $O(n \log^2 n)$-time algorithm that outputs a disjoint connection between $A$ and $B$. 

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Ham-Sandwich cut

A (disjoint) connection
Question 2:  Given a set $P$ of $n$ points in the plane in general position, and given an angle $\theta$, project the points orthogonally onto a directed line at angle $\theta$. The resulting order of the projections is called an allowable permutation. (Let us assume that no two points project to the same point.)

1. Prove that there are $O(n^2)$ distinct allowable permutations.

2. You are given two sets of points $B$ and $R$ (called, blue and red, respectively), each of size $n$. Give an $O(n^2 \log n)$-time algorithm which determines whether there exists a direction $\theta$, such that the orthogonal projections of the points of $B \cup R$ onto a line in this direction alternates between blue and red.

Question 3:  Consider a collection of $n$ points $P$ in the plane. Define a 3-slab to be the region bounded by a pair of non-vertical parallel lines, such that there are at least 3 points in the region (including on the lines). Define the height of a 3-slab to be the vertical distance between its two lines ($h$ in the picture below). Present an $O(n^2)$-time algorithm which computes a 3-slab of minimum height.