Strongquest moving knives

$n = 3$ empty tree

1. Ref moves knife to the right
2. Each player moves his/her knife to the right

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  X   L_i   R_i
  R   P_i
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**Strategy:**

\[ V_i(L_i) = V_i(R_i) \]

3. Player X calls STP, receives X
   + He/middle player's knife also cuts, producing \( Y \) and \( Z \)

\( P_i \)
Say \( P_x \) gets \( X \) as the other 2 players the one whose knife is closest to the knife gets \( Y \). The other player gets \( Z \).

We will show this is envy-free (and therefore proportional).

1) The player who stops \( P_x \) knows \( P_x \) gets \( X \) and knows the other 2 pieces are \( Y + Z \). \( P_x \) cannot envy any body - \( P_x \) knows \( X \) over \( Y + Z \).

2) The other players must think \( Y \) or \( Z \) is better than \( X \) since they did not say stop.

\( P_y \) receives \( Y \) so either \( P_y \)'s knife is left of \( P_y \)'s (c) or not (d).

![Diagram]

\( v_y(Y) = v_y(Z) \)

\( P_y \) gets \( Y \) and thinks it is bigger than \( X \).

\( P_y \) now cares.
b) Value of rest of cake

\[ v_y(Y) = v_y(\{2\}) > v_y(\{X\}) \]

\[ \therefore P_y \text{ cannot envy} \]

3) \( P_2 \text{ either was \textit{wiser} or not} \)

\[ v_{x\{2\}}(Y) = v_{x\{2\}}(\{2\}) > v_{x\{2\}}(X) \]

\[ P_2 \text{ cannot envy} \]

or

\[ v_{x\{2\}}(Z) \geq v_{x\{2\}}(Y) > v_{x\{2\}}(X) \]

\[ P_2 \]

\[ \text{No envy!} \]