Share like a pirate

Modified Dubui's Spanier (thanks: Troy, Mike)

Austin \( \frac{1}{m} \) (reprise)

Divide a cake into \( m \) equal pieces, as seen by \( n \geq 2 \) players

Trimming Algorithm (Moving Cake)

Some other non-continuous methods

Line choosers (people keep joining the division problem)

\[ \text{Austin} \]

Recall: \( n \) agree on a piece that is \( \frac{1}{m} \) in value

(continued on next page)

Pirate Ship:

<table>
<thead>
<tr>
<th>Captain</th>
<th>agreed upon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surgeon</td>
<td>200</td>
</tr>
<tr>
<td>Carpenter</td>
<td>100</td>
</tr>
</tbody>
</table>

Injuries:

| Right arm | 600 |
| Left arm  | 500 |
| Right leg |     |
| Left leg  | 400 |

\[ \text{Captain} \quad 6 \quad \text{Crew (Adult)} \quad 1 \]

\[ \text{Master} \quad 4 \quad \text{Crew (juvenile)} \quad \frac{1}{2} \]

New instance: divide remaining piece as if it is a new cake.

\[ \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \]
$n = 6$

$0 - 5 \quad 5$

$V_1(\pi) = \frac{1}{3} \quad V_2(\pi) = \frac{1}{3} \quad V_3(\pi) = \frac{1}{3}$

New cake

Recall: 2 agree on piece that is $\frac{1}{m}$ in value.

$b$ looks at markings & may agree 1 piece $\frac{1}{m}$ $V_b(\pi) = \frac{1}{m} = V_b(\pi)$

If not, no piece has exactly $\frac{1}{m}$ in value to $b$.

Some less, others more.

Can't all be same.

Can't all be more.

$b$ votes for $P_2$.

2 pieces, designated by $P_1$ & $P_2$.

$b$ says stop when also sees $\frac{1}{m}$ in value.
Ex. Find $y$ of origin, then $\frac{y}{m} = \frac{1}{3}$...

Now $P$ and $B$ repeat process, looking for $\frac{1}{m}$ of $AC$ captive

Result has $\frac{1}{m}$ value of origin, came to both,

**Divide into 4 equal pieces**

Apply **knife** (moves discretely)

\[
\begin{align*}
&\{1\} \quad \{2\} \\
&\{3\} \quad \{4\} \\
&\{5\} \quad \{6\} \\
&\{7\} \quad \{8\}
\end{align*}
\]

$P_1$ did not say stop — above the cut is $\frac{1}{2}$, below is more (for $P_2$)

$P_1$ moves items either side of the cut, always keeping $\frac{1}{2} \& \frac{1}{2}$

Then $P_2$ says stop — some sort of agreement (not perfect)
Baron - Master

Moving Gate / Last Diminisher / Thinning Alg (precedes moving knife)

Works in rounds

\( n \) parties

\( n \) rounds - in each round, 1 party receives cake

\[ \begin{array}{c}
\text{Parties} \\
\text{Cake} \\
\text{trimmings bucket}
\end{array} \]

Rules:

1) In round \( r \), when person \( P_i \) receives cake \( x_{r,i} \), then \( P_i \) creates \( x_{r+1,i} \) by trimming away \( d_i \), \( d_i \geq 0 \)

2) In round \( r \), the last party \((n)\) to create \( d_{r,n} \), \( d_{r,n} > 0 \) receives \( x_{r,n+1} \)

3) In round \( n+1 \) (2 parties left), do divide & choose

Strategy:

\[ x_{r,i} \xrightarrow{} x_{r+1,i} \]

\[ d_r \]

Strategy to guarantee proportionality

If \( \forall i \geq 1, \sum_{i=1}^{n} x_{r,i} = x_{r,n} \)

else \( x_{r+1,n} = x_{r,n} \)

In round \( 1 \), \( x_{1,1} = \text{whole cake} \)

In round \( c = 1 \), \( x_{c,i} = 0 \)

all \( c \leq n \)
**Example:**

1. **Preferences:**
   - Uniform
   - Likes left side
   - Likes right side

2. \( X_{12} \rightarrow \) Snip \( x_{12} \) left
   - \( V_a(x_{12}) = \frac{1}{2} \)
   - \( V_a(x_{13}) = \frac{1}{3} \)
   - \( V_a(d_{12}) = \frac{1}{6} \)

3. \( x_{13} \rightarrow \) Snip \( x_{13} \)
   - \( x_4 = x_{14} \)
   - \( V_3(x_{14}) = \frac{1}{4} \)
   - \( V_3(\text{rest of cake}) = \frac{1}{4} \)

**Round 1:** \( X \) is given to 2 - out

- \( P_1 + P_3 \) divide and choose
- \( V_r(\text{rest of cake}) \geq \frac{2}{3} \)
- \( V_3(\text{rest of cake}) \geq \frac{7}{9} \)

Suppose \( P_1 \) divides each \( \geq \frac{1}{3} \)

- \( P_1 \rightarrow \frac{1}{3} \)
- \( P_2 \rightarrow \frac{1}{3} \)

One piece must \( V_3(x) \geq \frac{1}{3} \cdot \frac{8}{9} = \frac{8}{27} \)

- \( \frac{8}{27} = \frac{2}{3} \cdot \frac{1}{3} \)

**Example was proportional:** Is it always?

Each piece after \( i \) has value \( \leq \frac{1}{i} \) to \( i \)

- Eventually, since cake has value 1, I will see a piece \( \geq \frac{1}{2} \)
- We will get that piece if last diminisher

** envy-free? No!** (earlier drop-out might envy someone who got their piece later)

- Last player envies nobody, last piece must have value \( \geq \frac{1}{2} \) (the dude's choice)
- Show same is true for penultimate
Regular Auction vs. Dutch Auction

my value = 10

- 6 I wh
- 5
- 1