Corrections to and Discussion of “Implementation and Evaluation of Mixed-criticality Scheduling Approaches for Sporadic Tasks”

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The AMC-IA mixed-criticality scheduling analysis was proposed as an improvement to the AMC-MAX adaptive mixed-criticality scheduling analysis. However, we have identified several necessary corrections to the AMC-IA analysis. In this article, we motivate and describe those corrections, and discuss and illustrate why the corrected AMC-IA analysis cannot be shown to outperform AMC-MAX.

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1. INTRODUCTION

The AMC-IA mixed-criticality scheduling analysis [Huang et al. 2014] was proposed as an improvement to the AMC-MAX adaptive mixed-criticality scheduling analysis [Baruah et al. 2011]. AMC-IA uses two definitions of a function \( n(s) \) to represent the number of releases of a task by time \( s \) which is defined as the last deadline before a criticality change. For low-criticality tasks, \( n \) is defined by:

\[
 n_j(s) = \left\lfloor \frac{s}{T_j} \right\rfloor. \tag{1}
\]

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For high-criticality tasks, a different definition for $n$ is given:

$$n_k(s) = \max \left( \left\lfloor \frac{s - D_k}{T_k} \right\rfloor + 1, 0 \right).$$  \hfill (2)$$

Note that subscript $j$ is used in the first definition and $k$ in the second.

The response-time of a two-criticality system, during the change from low-criticality mode LO to high-criticality mode HI (0 to 1), is given by the following equation (from Huang et al. [2014])

$$R_s^i = C_i(0) + \sum_{\tau_j \in H_i} n_j(s)C_j(0) + \sum_{\tau_k \in HHC_i} \left( \left\lceil \frac{R_s^i}{T_k} \right\rceil - n_k(s) \right) C_k(0),$$  \hfill (3)$$

where $H_i$ is the set of task with higher priority than $\tau_i$ (of either criticality) and $HHC_i$ is the set of task with higher priority and higher (or equal) criticality than $\tau_i$.

2. CORRECTIONS TO AMC-IA

The first correction to the AMC-IA analysis is that the initial computation time (for $\tau_i$) itself should be $C_i(1)$ as it executes for its maximum value. The second correction to AMC-IA, to remove confusion as to which of the $n$ functions should be used, is to rewrite the preceding equations in an equivalent but more obvious form. We still define $n$ by Equation (1) and introduce a new function $m$ to encode Equation (2):

$$m_k(s) = \max \left( \left\lfloor \frac{s - D_k}{T_k} \right\rfloor + 1, 0 \right).$$  \hfill (4)$$

We then rewrite Equation (3):

$$R_s^i = C_i(1) + \sum_{\tau_j \in HLC_i} n_j(s)C_j(0) + \sum_{\tau_k \in HHC_i} m_k(s)C_k(0) + \sum_{\tau_k \in HHC_i} \left( \left\lceil \frac{R_s^i}{T_k} \right\rceil - m_k(s) \right) C_k(1),$$  \hfill (5)$$

where $HLC_i$ is the set of low-criticality tasks with higher priority than $\tau_i$, and $HHC_i$ is the set of high-criticality tasks with higher priority than $\tau_i$.

Following this notational clarification, we then modify the definition of AMC-IA itself. As the third correction, the “+1” is removed from Equation (4):

$$m_k(s) = \max \left( \left\lfloor \frac{s - D_k}{T_k} \right\rfloor , 0 \right).$$  \hfill (6)$$

The fourth correction to the AMC-IA analysis is that each “s” point is now defined to be the first deadline after a criticality mode change.

3. DISCUSSION

The two significant corrections described earlier are to the definitions of “s” and the equation for $m_k(s)$. The change to “s” is necessary because the initial definition could underestimate the impact of high-priority low-criticality tasks on other tasks. This is easy to see with a simple system that has one high-priority low-criticality task $\tau_1$ with $D_1 = T_1 = 10$, and a set of high-criticality tasks with $D > 12$. If all tasks are released at time 0, and a mode change occurs at time 12 (in some high-criticality task), then the

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1In Huang et al. [2014], LO and HI criticalities are denoted as 0 and 1, respectively; thus, for example, the LO-criticality WCET of $\tau_i$ is denoted $C_i(0)$ (rather than $C_i(LO)$ – the notation used in Baruah et al. [2011]).
Corrections to and Disc. of “Impl. and Eval. of M-C Scheduling Approaches for Sporadic Tasks” 77:3

“last deadline before 12” is at time 10; Equation (1) gives $n(10) = 1$, but it should be 2, as releases at times 0 and 10 will impact tasks at time 12. To avoid underestimating the interference given by Equation (1), it is necessary to define “s” as the first deadline after a criticality mode change. Unfortunately, this can now underestimate the interference of high-criticality tasks, as illustrated in Figure 1.

In this example, consider a high-criticality task $r_2$ with $C_2(LO) = 2$, $C_2(HI) = 3$, $D_2 = 3$ and $T_2 = 10$. If a criticality mode change occurred at time 4 (with all tasks released at time 0, and all other tasks having deadline later than time 13), then the new definition of “s” would give a point at time 13 (as 13 is the next deadline after time 4). At this point, Equation (4) gives a value of 2. This implies that in any interval after 13 there will be two executions of the task with $C(LO)$. It is easy to see that this is wrong, as at time 13 there may be only one execution with $C(LO)$ and hence one with $C(HI)$. As $C(HI) \geq C(LO)$ this could lead to an underestimation of this task's interference. To correct for this error, the “+1” in Equation (4) is removed to give Equation (6). Now $m(13) = 1$, which is a sufficient correction.

If these two corrections are applied, then it is not clear how Equation (5) can lead to tighter analysis than AMC-MAX. With the removal of “+1,” a task with $D = T$ will assume no $C(LO)$ interference unless “s” is $2T$ or greater. AMC-MAX will assume (for $s < 2T$) either 0 or 1 $C(LO)$ hits - depending on the value of $R$. Hence, they will often give the same result, but AMC-MAX can in some situations deliver (correctly) less interference.

It remains an open question whether there is a definition of AMC-IA that lies between the incorrect published one and the one given earlier that is both sufficient and “better” than AMC-MAX, where “better” means that it will deem more task sets to be schedulable. Certainly AMC-MAX is not exact, so such a scheme is possible. However, with our current investigation, it appears so far that in order for AMC-IA to be made sufficient it may not be able to outperform AMC-MAX.

4. CONCLUSIONS

In this article, we have described two necessary corrections to the AMC-IA analysis and have shown that with those corrections AMC-IA cannot be shown to outperform AMC-MAX. It would be helpful to examine further whether the approach taken by AMC-IA offers potential insights into how AMC-MAX could be improved, though as we have demonstrated in this article, such improvement remains future work.

REFERENCES

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