1. **Kleinberg & Tardos** Chapter 5, question 3

2. Dimensionality reduction is a widely-used technique for accelerating computations on high-dimensional data, by replacing them with a lower-dimensional “summary” that preserves the key properties of the original data, typically the approximate distances of points. The key step in fast dimensionality reduction is multiplying a vector by a matrix from the following family of special matrices, $H_0, H_1, \ldots, H_k, \ldots$:

   - $H_0$ is the $1 \times 1$ matrix $[1]$.
   - For $k > 0$, $H_k$ is the $2^k \times 2^k$ matrix
     \[
     H_k = \begin{bmatrix} H_{k-1} & H_{k-1} \\ H_{k-1} & -H_{k-1} \end{bmatrix}
     \]

What makes the method so fast is that, for a vector $v$ of length $n = 2^k$, the matrix-vector product $H_k v$ can be computed in time $O(n \log n)$. Show how to do this (and as usual, prove the correctness and time bound of your algorithm).

3. **Kleinberg & Tardos** Chapter 4, question 4