

# Parallel Determinacy Race Detection for Futures

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## Abstract

The use of futures can generate arbitrary dependences in the computation, making it difficult to detect races efficiently. Algorithms proposed by prior work to detect races on programs with futures all have to execute the program sequentially. We propose F-Order, the first known parallel race detection algorithm that detects races on programs that use futures. Given a computation with work  $T_1$  and span  $T_\infty$ , our algorithm detects races in time  $O((T_1 \lg \hat{k} + k^2)/P + T_\infty(k + \lg r \lg \hat{k}))$  on  $P$  processors, where  $k$  is the number of future operations,  $r$  is the maximum number of readers per memory location, and  $\hat{k}$  is the maximum number of future operations done by a single future task, which is typically small. We have also implemented a prototype system based on the proposed algorithm and empirically demonstrates its practical efficiency and scalability.

\*CCS Concepts • **Software and its engineering** → **Software testing and debugging**; • **Theory of computation** → *Shared memory algorithms*; • **Computing methodologies** → *Shared memory algorithms*.

## 1 Introduction

Futures [17] provide an elegant means to express parallelism in functional programs. Using futures, the programmer can annotate the invocation of a function instance to be asynchronous – the function call may execute in parallel with the continuation of the caller. The invocation returns a **future handle** object that associates the function call execution, the **future task**, with a store. Whenever the execution of a future task completes, the resulting value of the task is deposited into the future handle and can be retrieved by anyone who has access to the handle. An attempt to retrieve the result from the handle before the computation finishes causes the control to block.

Although futures were originally proposed in the context of functional programs, its use has seen an increasing popularity in the context of imperative programs. Modern imperative task parallel platforms that support futures include variants of Habanero Java [9, 21], X10 [10, 48], Cilk [43], and Deterministic Parallel Ruby [27]. Its use has also been incorporated into C++11 [22] and Java [34]. Recently, researchers have also studied how to use futures in multithreaded computations to share concurrent data structures [24, 30].

Compared to fork-join parallelism, which is the traditional parallelism paradigm supported by many task parallel platforms, futures offer additional flexibility in how one can express parallelism. With fork-join parallelism, the asynchronous function invocations must be joined together within a well-defined lexical scope; with futures, joining with a future task occurs when one invokes **get** on the handle to attempt to retrieve the result of the future task. Since the future handle is an object that can be freely passed around, or even stored in a data structure and retrieved later, joining with future tasks can occur at arbitrary program points and is not limited by any lexical scope.

With imperative programming, the interactions between shared memory and parallelism have to be regarded carefully, however. If not careful, the interactions can lead to pernicious errors such as races that are challenging to detect and debug. In this paper, we study the problem of how to efficiently detect races in a task parallel program that employs futures.

How to efficiently detect races in a task parallel program is a well studied problem [1, 4, 13–15, 29, 37, 38, 47, 50, 51, 55]. In the context of task parallel programming, the focus has been on detecting **determinacy races** [13] (also called **general races** [31]), where two *logically* parallel subcomputations access the same memory location in a conflicting way (i.e., at least one is a write). The execution of a parallel computation can be modeled as a **directed acyclic graph** (or **DAG**), in which a node represents a sequence of instructions without parallel control constructs and an edge represents a control dependence that arises from the execution of control constructs. Two memory accesses are said to be logically in parallel if no directed path exists between them. Since the dependences that arise from the control constructs are input-dependent and not schedule dependent, the race detection algorithms proposed by this prior work can provide strong correctness guarantees – the race detector reports a race if and only if one exists for a given program and a given input; i.e., the race detector is *sound and complete with respect to a given input*, as opposed to for a given execution

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schedule. Moreover, in the absence of a determinacy race,<sup>1</sup> the computation behaves in a deterministic fashion.

Algorithms proposed by this prior work detect races *on the fly* as the program executes. They typically consist of two components: 1) an *access history* that keeps track of the readers and the writers that previously accessed a given memory location during execution, and 2) a *reachability* component that answers the query of whether two given accesses are logically in parallel or not. Upon a memory access  $v$ , the algorithm checks  $v$  against each conflicting access  $u$  stored in the access history that accessed the same memory location, and queries the reachability data structure to see if  $u$  and  $v$  are logically in parallel. If so, the algorithm has found a race.

Much of the prior work focuses on detecting races for fork-join parallelism [4, 13–15, 29, 37, 38, 50], which generates computation DAGs with nice structural properties. By exploiting the structural properties, the state-of-the-art algorithm [50] can detect races for a task parallel computation with only fork-join parallelism with no asymptotic overhead. Given a computation with *work*  $T_1$ , how long it takes to run the computation on one core and *span*  $T_\infty$ , how long it takes to run the computation on infinitely-many cores, or the longest dependences in the computation, the algorithm can race detect the computation in time  $O(T_1/P + T_\infty)$  when running on  $P$  processing cores.

Unlike computations that utilize only fork-join parallelism, the use of futures can generate arbitrary dependences due to the fact that the joining of future tasks can occur at arbitrary program points. Consequently, a program that uses futures no longer has the same structural properties, and algorithms developed in this prior work cannot be applied directly.

A few *sequential* algorithms [1, 47, 51] have been proposed to race detect programs that use futures, and the state-of-the-art algorithm [1] provides an execution time bound of  $O(T_1 + k^2)$ , where  $k$  is the number of future operations. However, these prior algorithms must execute the program sequentially during race detection. The requirement of sequential execution is not just an implementation artifact but fundamental to how the algorithms maintain reachability. As the computation DAG unfolds dynamically during program execution, the reachability maintenance data structure must maintain reachability of all accesses occurred thus far. In these prior algorithms, the reachability data structure can only be maintained correctly assuming a particular traversal order of the computation DAG, which only the sequential execution guarantees. If the DAG folds in any other topological traversal order (which occurs during parallel executions), the reachability data structure proposed by these prior algorithms no longer work correctly.

In this work, we propose F-Order, the first known *parallel* race detection algorithm that race detects a program with futures while executing the program in parallel. Given a computation with  $T_1$  work and  $T_\infty$  span, our race detection algorithm runs in time  $O((T_1 \lg \hat{k} + k^2)/P + T_\infty(k + \lg r \lg \hat{k}))$  on  $P$  processors, where  $k$  is the number of future operations,  $r$  is the maximum number of readers per memory location, and  $\hat{k}$  is the maximum number of future operations done by a single future task, which is usually small.

To put this bound into perspective, a provably efficient parallel scheduler can execute a baseline program (i.e., no race detection) in time  $O(T_1/P + T_\infty)$  [3]. Our race detection algorithm incurs additional  $O(k^2)$  work, like the state-of-the-art sequential algorithm [1], but this additional work can be parallelized. It also incurs a multiplicative overhead of  $\lg \hat{k}$  on the work term (which is small) and a multiplicative overhead of  $(k + \lg r \lg \hat{k})$  on the span term, where  $k$  likely dominates the other term.

We have implemented and empirically evaluated a prototype system based on F-Order. The empirical results indicate the reachability component incurs little overhead and that the race detection obtains similar scalability as the baseline. Moreover, we have compared our parallel race detector against FutureRD, the state-of-the-art sequential race detector for futures [49, 51]. Empirical results indicate that, even though our parallel race detector incurs higher overhead on one-core execution, the fact that we can race detect while executing the program in parallel quickly pays off in absolute execution times.

## Contributions

- We propose F-Order, the first known parallel race detection algorithm that race detects programs with futures. We provide the key intuitions behind our parallel algorithm (Section 4) and prove its correctness (Section 5).
- We show that F-Order can race detect a computation with work  $T_1$  and span  $T_\infty$  in expected time  $O((T_1 \lg \hat{k} + k^2)/P + T_\infty(k + \lg r \lg \hat{k}))$  on  $P$  processors, where  $k$  is the number of future operations,  $r$  is the maximum number of readers per memory location, and  $\hat{k}$  is the maximum number of future operations done by a future task (Section 6).
- We implemented a prototype of F-Order and empirically evaluate its scalability and overhead (Section 7). Experimental results indicate that F-Order scales well and the absolute running times of all benchmarks with race detection using F-Order on 4 cores or more beat the state-of-the-art sequential race detector [51].

## 2 Preliminaries

**Language Constructs Considered.** Fork-join parallelism can be expressed using two keywords: `spawn` and `sync`. When a function  $F$  *spawns* off another function  $G$  by prefixing the invocation with `spawn`, the execution of  $G$  may

<sup>1</sup>Henceforth, when we say a race, we mean a determinacy race unless noted otherwise.

operate in parallel with the continuation of  $F$ . The invocation of a sync specifies that all previously spawned functions must return before the control can pass sync.<sup>2</sup>

Future parallelism can be expressed using create and get. Similar to spawn, when a function  $F$  spawns off another function  $G$  by prefixing the invocation with create, the execution of  $G$  may operate in parallel with the continuation of  $F$ . We refer to this instance of  $G$  as a **future task**. Unlike spawn, however, the invocation of create returns a **future handle**, with which the execution of the future task  $G$  is associated. By invoking a get on the handle, the control cannot pass beyond get until the future task finishes and deposits its result into the handle. An invocation of a sync does not affect future tasks spawned off with create; sync can freely continue without waiting for future tasks to return.

**Modeling Parallel Computations.** One can model the execution of a parallel computation as a **directed acyclic graph** (or **DAG**), in which a node represents a sequence of instructions without parallel control and an edge represents a control dependence between two nodes.

The execution of a spawn creates a **spawn node** with two outgoing edges: one to the spawned function (drawn as the left sub-DAG) and one to the continuation of the caller (drawn as the right sub-DAG). The execution of a sync creates a **sync node** with multiple incoming edges, one from the end of each spawned function to the sync node that represents the continuation after sync. Without loss of generality, we shall assume that a sync code consists of only two incoming edges — it is not difficult to convert a sync node with multiple incoming edges into a chain of sync nodes, each with two incoming edges.

The execution of a program that uses only fork-join parallelism generates a **series-parallel (SP) DAG** [52] with nice structural properties. Specifically, a SP DAG is a planar DAG with a unique **source node** with no incoming edges and a unique **sink node** with no outgoing edges. A SP DAG can be constructed recursively using series and parallel compositions where the spawn and sync nodes are well-nested.

The execution of a create generates a **create node** with two outgoing edges, one to the future task (drawn as the left sub-DAG) and one to the continuation of the caller (drawn as the right sub-DAG). The execution of a future task terminates with a **put node**, which is the last node to execute in the future task that deposits the result into its future handle. The execution of a get terminates the current node  $v$ , and generates a **join node** with two incoming edges — one from  $v$ , referred to as join node's **local parent**, and one from the put node of the corresponding future task, referred to as the join node's **future parent**. We refer to the edge generated by create to the future task and the edge generated by get from the put node to the join node as **non-SP edges** as these

edges do not conform to the well-nested structures that arise from series and parallel compositions for SP DAGs.

When a program uses both fork-join and future parallelism, the execution generates a **nearly series-parallel (NSP) DAG**  $G_N = (D_{sp}, E_{non})$ , a DAG formed by a set  $D_{sp}$  of SP DAGs connected by a set  $E_{non}$  of **non-SP** edges. Since each future task may contain fork-join parallelism, the execution of a future task can be modeled as its own SP DAG. If the future task executes a create, the spawned-off future task is a separate SP DAG, connected via non-SP edges.

**Other Definitions.** The following types of nodes are special: spawn, sync, create, join, and put nodes. All other nodes are **common nodes**. Given two nodes  $u$  and  $v$ , we say  $u$  and  $v$  are **in series** if a directed path exists between  $u$  and  $v$ ; otherwise they are **in parallel**. If a path exists between  $u$  and  $v$ , we say  $u$  is an **ancestor** of  $v$ , and  $v$  is a **descendent** of  $u$ . Furthermore, we say create and put nodes are **non-SP nodes**, which are special in that they have an outgoing non-SP edge. If there is a path from  $u$  to  $v$ , and  $u$  is either a create or put node, we say  $u$  is a **non-SP ancestor** of  $v$ .

**Parallel Schedule and Performance Metrics.** During execution, the DAG unfolds dynamically, and the scheduler maps execution of nodes onto processing cores, in a way that respects the dependences expressed by the parallel control constructs. A node  $v$  only becomes ready to execute when all its ancestors have finished executing. A valid parallel schedule is a topological sort of the nodes in the DAG.

There are two important metrics for measuring performance of a DAG: assuming each node takes unit time to execute, its **work** (denoted as  $T_1$ ), the number of nodes in the DAG, or how long it takes to execute the DAG on one core; and its **span** (denoted as  $T_\infty$ ), the length of a longest path in the DAG, or how long it takes to execute the DAG on infinitely many cores. A DAG can be scheduled efficiently using a **work-stealing** scheduler [2, 3, 7, 8]. Given a DAG with  $T_1$  work and  $T_\infty$  span, a work-stealing scheduler can execute the computation in time  $T_1/P + O(T_\infty)$ .

### 3 Related Work

**Race Detection for Task Parallel Code.** One can exploit the structural property of fork-join parallelism to obtain efficient race detection algorithms when the program uses only fork-join parallelism. Mellor-Crummey [29] is the first to show that, for a parallel race detector, it suffices to store two readers and a single writer per memory location in the access history. Feng and Leiserson [13, 14] provided the first provably efficient sequential race detection algorithm that is near-optimal. Bender et al. [4] proposed the first asymptotically optimal sequential algorithm and the first provably efficient parallel algorithm with a  $O(P)$  overhead on the span term, where  $P$  is the number of cores used during execution. Finally, Utterback et al. [50] proposed the first asymptotically optimal parallel algorithm.

<sup>2</sup>Many task parallel platforms also support parallel loops, which can be thought of as syntactic sugar that compiles down to spawn and sync.

For race detecting pipeline parallelism, which is another task parallel paradigm with nice structural properties, Dimitrov et al. [12] proposed the first sequential algorithm that is near-optimal; Xu et al. [55] later proposed the first asymptotically optimal parallel algorithm.

Lee and Schardl [25] proposed a sequential race detection algorithm for fork-join computations with reductions, where the DAG is series-parallel except when executing reductions.

A few algorithms [1, 47, 51] have been proposed to race detect programs that use futures. Surendran and Sarkar [47] proposed the first algorithm to race detect a program that uses futures, but their algorithm runs sequentially and can incur large overhead,  $O(T_1(f+1)(k+1)\alpha(m,n))$ , where  $f$  is the number of future tasks,  $k$  is the number of future operations,  $m$  is the number of memory accesses,  $n$  is the number of parallel control constructs executed, and  $\alpha$  is the functional inverse of Ackermann's function. Later, Agrawal et al. [1] improved the bound to  $O(T_1+k^2)$ , although the work is theoretical and no implementation of the algorithm exists. Finally, Utterback et al. [51] separated the use of futures into two classes — **structured** use of futures that imposes certain programming restrictions on where the future get can occur,<sup>3</sup> and **general** use of futures that does not impose such a restriction. By distinguishing the two, Utterback et al. [51] observed that programs that use structured futures can be race detected much more efficiently, in time  $O(T_1\alpha(m,n))$ . For general use of futures, Utterback et al. [51] proposed an algorithm (and its corresponding implementation) that executes in time  $O((T_1+k^2)\alpha(m,n))$ .

**Race Detection for Pthreaded Code.** Much work has been done on race detecting pthreaded code. Like task parallel code, an execution of a pthreaded code can also be represented as a DAG and thus the problem of race detection boils down to checking for reachability of nodes in the DAG. Unlike task parallel code, however, the dependences in a pthreaded execution are formed via lock operations (as opposed to via high-level constructs) whose ordering forms **happens-before (HB) relations**. Thus, the dependences are *schedule* dependent and nondeterministic for a given input. Thus, race detecting pthreaded code cannot be sound and complete for a given input as the number of possible schedules grows quickly.

The dependences formed via lock operations can be arbitrary with no structural properties. Researchers have proposed lock-set based algorithms [41, 53] that provide wide coverage but cannot precisely capture the dependences and may generate many false positives. Researchers have also proposed a Vector-clock (VC) based approach [16]) that precisely captures the dependences, but only for a given schedule. Hybrid approaches have also been explored [33, 36, 42, 56] that incorporate VC and lock-sets in order to achieve a

compromise between coverage and precision. Finally, predictive analysis has been proposed as a method to explore alternative feasible schedules among nearby instructions in order to maintain precision while increasing the coverage [23, 26, 40, 44].

Even though a VC-based algorithm can capture arbitrary dependences and in principle can be applied to task-parallel code with futures, naively applying it to task-parallel code would be impractical. It requires storing a vector-clock of length  $n$  with each memory location, where  $n$  is the number of nodes in the computation DAG (which can be on the order of millions). Each memory access requires querying this vector, resulting a  $O(n)$  overhead per access. In contrast, for task parallel code, one can exploit the structural properties in the DAG to encode reachability much more efficiently. In the case where futures are used, our algorithm still exploits the structural properties that exist within a single future task (which is an SP DAG) to allow for efficient reachability queries and handles the arbitrary dependences that arise due to non-SP edges in a special way. By doing so, our algorithm incurs an additional  $O(k^2)$  space overhead across the entire execution and additional  $O(\lg \hat{k})$  work per access, where  $k$  is the number of future operations, and  $\hat{k}$  is the maximum number of future operations done by a single future task.

**Race Detection for Event-Driven Applications.** More recently, researchers have begun to study algorithms to race detect event-driven systems, such as web pages [35, 39] and mobile applications [5, 19, 20, 28]. A key challenge here, unlike the pthreaded code, is to establish a precise **causality model**, i.e., how to capture happens-before relations between asynchronous events. These prior works mainly differ in their formulations of the causality model, which can be specific to the application domain (i.e., a model for web applications written in JavaScript will differ from that for Android applications). Similar to pthreaded code, once the causality model is established, an execution of an event-driven system can be represented as a DAG with arbitrary dependences, and the problem of race detection boils down to checking for reachability of nodes in the DAG.

Most prior work in this domain performs reachability queries by building and traversing the DAG explicitly [5, 20, 28, 35, 39], which can incur high overhead either during the DAG construction (i.e., the causality model requires traversing through the execution trace a few times to finalize the DAG) or during detection. Work by [39] builds the DAG but improves the overhead by running a VC-based algorithm on the DAG using “chain decomposition,” where the width of a vector is determined by the number of chains in the DAG. Work by [5] further improves the overhead by proposing a causality model where the DAG can be constructed with almost a single pass of the execution trace. Finally, work by [19] proposes a causality model that allows

<sup>3</sup>The structured use of futures is first proposed by Herlihy and Liu [18] in the context of studying cache misses.

one to run a VC-based algorithm (with chain decomposition) with single-pass over the trace without building the DAG explicitly. This line of work differs from our work in that, the causality model (or how dependences form) for task parallel code with futures is much simpler, allowing us to encode reachability without explicitly building the DAG. While some work [5, 19, 39] proposed heuristics to optimize a VC-based algorithm, the optimizations are domain specific and not applicable in our case.

**Use of Futures.** Surendran and Sarkar [46] propose compiler analysis to automatically generate parallel code from serial code for pure functions using futures. Voss et al. [54] investigated methods to detect (and prevent) deadlock in applications that use futures. Finally, researchers have studied scheduling strategies and the corresponding performance bounds when a task parallel code uses futures [18, 43, 45].

## 4 Overview of F-Order

This section provides an overview of F-Order. The use of futures generates non-SP edges (defined in Section 2) that form arbitrary dependences and thus lack the structural properties that fork-join parallelism enjoys. This has two implications. First, it no longer suffices to store only a constant number of accessors per memory location in the access history. Second, the reachability can no longer be maintained efficiently by exploiting the structural properties. We discuss each of the components in turn, the intuitions behind F-Order, and how F-Order addresses the challenges.

### 4.1 Access History in F-Order

For fork-join parallelism, due to the nice structural properties of SP DAGs, it is sufficient to store only the “left-most” and “right-most” readers per memory location during parallel execution [29]. One can prove that a reader omitted by the access history can race with a writer if and only if the writer also races with either the left-most or the right-most reader. Thus, we do not miss a race by omitting such a reader in the access history. For a program that uses futures, however, we no longer have the same structural properties — there are no clear “left-most” and “right-most” readers that one can store to subsume potential races with other readers. Thus we must store all readers encountered until a sequential writer comes along.

How F-Order maintains access history follows the same strategy as the prior state-of-the-art sequential algorithm [1]. For each memory location  $l$ , F-Order stores a *last writer*,  $\text{last-writer}(l)$ , which is the last node that wrote to  $l$ , and a list of readers  $\text{reader-list}(l)$  that read  $l$  since  $\text{last-writer}(l)$ . Whenever a node  $r$  tries to read a memory location  $l$ , F-Order performs a reachability query between  $r$  and  $\text{last-writer}(l)$  to see if  $r$  races with the last writer. If so, a race is reported. If not,  $r$  is added to

$\text{reader-list}(l)$ . Whenever a node  $w$  writes to a memory location  $l$ , F-Order checks  $w$  against all the readers in  $\text{reader-list}(l)$  and the last writer  $\text{last-writer}(l)$ . If any of the  $\text{reader-list}(l)$  or  $\text{last-writer}(l)$  is logically in parallel with  $w$ , a race is reported. Otherwise, F-Order empties the  $\text{reader-list}(l)$  and sets  $\text{last-writer}(l)$  to be  $w$ . As argued by Agrawal et al. [1], we won't miss any races by emptying  $\text{reader-list}(l)$  because any future access that races with a node in  $\text{reader-list}(l)$  must also race with  $w$ . The fact that the prior algorithm executes the program sequentially and F-Order executes it in parallel does not change the correctness argument, so long as F-Order synchronizes the access history data structure correctly.

Agrawal et al. [1] also show that that the total number of reachability queries per reader is bounded by two. F-Order provides the same bound on the number of reachability queries per reader. Since F-Order executes in parallel, however, we must also consider how the reachability queries impact the span, which we discuss in Section 6.

### 4.2 Reachability Maintenance in F-Order

**The Challenges.** A computation that uses futures can be modeled as a nearly-series-parallel DAG (NSP DAG), consisting of a set of SP DAGs connected via non-SP edges. Given two nodes, if they are connected by only SP edges, one can perform a reachability query on them efficiently by applying the reachability maintenance algorithm used for fork-join parallelism from prior work [4, 15, 50]. The challenge is to handle the reachability queries efficiently when the two nodes are possibly connected in part by non-SP edges.

The state-of-the-art prior algorithm [1] encodes the reachability that arises due to non-SP edges explicitly using an auxiliary graph  $\mathcal{R}$ . Unfortunately, the maintenance of  $\mathcal{R}$  heavily depends on traversing the computation DAG in a left-to-right depth-first fashion (i.e., executing the DAG sequentially). Thus, this prior algorithm simply cannot be parallelized, since a parallel execution can traverse the DAG in any order (as long as it is a topological sort of the DAG), which breaks the invariants required by  $\mathcal{R}$  to keep track of reachability correctly. Thus, the reachability maintenance in F-Order has to use an entirely different strategy.

**The Intuitions.** Based on how we model the computation, each future task forms its own SP DAG, and different SP DAGs can only be connected via non-SP edges. That means, if two nodes are in series and connected by only SP edges then they must belong to the same SP DAG, and one can utilize a prior parallel algorithm such as WSP-Order [50] to correctly answer reachability queries between them. WSP-Order cannot encode the reachability of two nodes correctly if they are connected via non-SP edges, regardless of whether they belong to the same SP DAGs or not. Thus, we need some other means to encode reachability between two nodes if they are connected via non-SP edges.

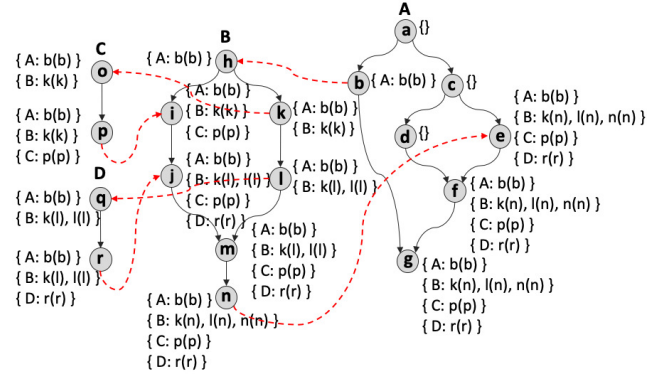
The key observation is as follows. Given two nodes  $u$  and  $v$  connected via non-SP edges, some node  $w$  must exist in the path between  $u$  and  $v$ , where  $w$  is a non-SP ancestor of  $v$  that is in the same SP DAG as  $u$ . That is, the prefix of the path (from  $u$  to  $w$ ) contains only SP edges, and the suffix (from  $w$  to  $v$ ) containing at least one non-SP edge (outgoing from  $w$ ). Thus, given two nodes  $u$  and  $v$  possibly connected in part by non-SP edges, we can query their reachability efficiently if we can quickly determine if such an ancestor  $w$  exists, who is 1) an non-SP ancestor of  $v$  in the same SP DAG as  $u$  and 2) in series with  $u$  via only SP edges.

**The FOM Data Structure.** To quickly determine whether such a non-SP ancestor  $w$  of  $v$  exists, F-Order employs an enabling data structure called the **Future Order-Maintenance** (or **FOM** for short) data structure per node in the NSP DAG. An FOM data structure for  $v$  stores all  $v$ 's non-SP ancestors, organized into groups, where each **group** contains non-SP ancestors from the same SP DAG.

Upon execution of a node  $v$ , its FOM data structure will be complete and its content fixed because  $v$ 's FOM data structure contains only  $v$ 's non-SP ancestors, and these must have already been discovered by the time  $v$  executes as the scheduler guarantees that a node cannot execute until all its ancestors have executed. If  $v$  accesses some memory location that some node  $u$  accessed previously, F-Order queries  $v$ 's FOM data structure to find the group  $g$  that holds the non-SP ancestors from the same SP DAG as  $u$ . F-Order then checks with  $g$  to determine whether some  $w$  exists that is reachable from  $u$ . If a node is in the group  $g$ , by definition it is in the same SP DAG as  $u$ . Then, we simply need to check if  $w$  is in series with  $u$ .

A naive implementation would be to check reachability against every node in  $g$ , taking time linear in the size of  $g$ . Ideally, we would like to quickly eliminate non-viable candidates in the group and avoid querying every single node in the group. The key insight of F-Order involves identifying the correct auxiliary data to store with each non-SP ancestor in a group so that the process of elimination can be done quickly.

It turns out that, to perform the process of elimination within a group, we simply need to organize nodes in a given group as follows. First, store the nodes in a total order, called the **English** order [32], that corresponds to the depth-first-left-to-right traversal of nodes in the corresponding SP DAG. The English order is well defined among nodes in the same group, because a group contains only nodes from the same SP DAG with no non-SP edges. Second, with each node  $w$  in the group, additionally store  $w$ 's **furthest descendant** that is also within the same group; that is, some node  $z$  that is reachable from  $w$  that is also in the group such that no other node  $y$  in the group is reachable from  $z$  (formally defined in Section 5). We will elaborate on the detailed construction of FOM data structures and discuss how F-Order uses them to perform reachability queries in Section 5.



**Figure 1.** An example of a NSP DAG with every node’s FOM data structure shown. In this NSP DAG, four SP DAGs exist, ID’ed as A, B, C, and D, with A being the main SP DAG and the others being the spawned future tasks. The non-SP edges are shown as thick dashed edges. Each node has its own instance of FOM data structure, containing entries of  $\{key : value\}$  pairs, where the  $key$  is the ID of an SP DAG and the corresponding  $value$  is a set of non-SP ancestors from the SP DAG. The parentheses next to each non-SP ancestor shows its furthest descendant in the group.

### 4.3 An Illustrating Example

Figure 1 shows the static snapshot of an NSP DAG with all its FOM data structures shown. Note that the parallel execution unfolds the NSP DAG dynamically, revealing each node as it becomes ready (i.e., all its ancestors have executed). Nevertheless, as discussed earlier, the content of an FOM data structure for node  $v$  is fixed by the time  $v$  executes, so this dynamic unfolding of the DAG does not change the content of the FOM data structures shown.

For now, we shall focus solely on the organization of an FOM data structure and how we use it to perform reachability queries. Take node  $f$ : it contains multiple non-SP ancestors. They are grouped into four entries in  $f$ 's FOM data structure, as they respectively belong to four different SP DAGs. In particular,  $f$  has nodes  $k$ ,  $l$ , and  $n$  as its non-SP ancestors, all from SP DAG B. Thus, its FOM data structure contains an entry with group keyed by B, and the corresponding value is a list of non-SP ancestors ordered in their English order. (Note that the nodes in a given SP DAG are labeled alphabetically according to their ordering in the English order.)

Say  $f$  is being executed, and F-Order wants to check if  $f$  is reachable from node  $i$ . Since  $i$  belongs to SP DAG B, F-Order checks  $f$ 's FOM data structure for the group indexed with B, which returns the list  $k$ ,  $l$ , and  $n$  (with every node having  $n$  as its furthest descendant). Since  $n$  is reachable from  $i$ , F-Order concludes that  $f$  is reachable from  $i$ . In this case, the group for B is small, containing only three nodes. However, the size of a group can be larger, and ideally we want F-Order to quickly home in on  $n$  and not check  $i$  against every node in the group. This is where the English ordering and the

auxiliary data of furthest descendants become useful, which we discuss in Section 5.

## 5 Details of F-Order and Its Correctness

This section presents the full detail of F-Order and its correctness proof. F-Order consists of two parts: a **construction** algorithm that builds and maintains the FOM data structure for each node and a **reachability-query** algorithm that checks whether a given pair of nodes are reachable from one another. We discuss each in turn. Throughout the section, we shall refer back to Figure 1 as an illustrating example.

**Notations.** Given an NSP DAG  $G_N = (D_{sp}, E_{non})$ , which consists a set of SP DAGs connected via non-SP edges, we assume each SP DAG  $d \in G_N$  is assigned with a unique integer identifier, denoted as  $SP(d)$ . Given a node  $u$ , we overload the notation and use  $SP(u)$  to denote the ID of the SP DAG containing  $u$ . Given two nodes  $u$  and  $v$ , we use  $u \rightsquigarrow v$  to denote the presence of a directed path from  $u$  to  $v$ . We use  $u \rightsquigarrow_{sp} v$  if the path comprises only SP edges and  $u \rightsquigarrow_{nsp} v$  if the path comprises any non-SP edge. We say  $u < v$  iff  $u \rightsquigarrow v$ , and  $u \leq v$  iff either  $u < v$  or  $u = v$ . If  $u$  and  $v$  are in the same SP DAG  $d \in D_{sp}$ , we say  $u <_d v$  iff  $u \rightsquigarrow_{sp} v$ ; we say  $u \parallel_{left}^d v$  iff node  $u$  is **left of**  $v$  in SP DAG  $d$ , where  $u$  is in the left sub-DAG of  $d$  and  $v$  is in the right sub-DAG of  $d$ . Both notations  $<_d$  and  $\parallel_{left}^d$  are only applicable to nodes in the same SP DAG  $d$ . They specify the English order in that, if  $u$  is before  $v$  in the order, then either  $u <_d v$  or  $u \parallel_{left}^d v$ . For instance, in Figure 1,  $c <_d g$  and  $b \parallel_{left}^d e$ . On the other hand,  $h$  and  $g$  cannot be related using these operators.

### 5.1 Construction of FOM Data Structures

As the NSP DAG unfolds, nodes become ready and get executed. When a node  $v$  executes, F-Order constructs an FOM data structure for  $v$ , denoted as  $v.fom$ , whose content is complete at the beginning of  $v$ 's execution to allow for reachability queries for memory accesses performed by  $v$ . An FOM data structure is organized as a hash table, hashing  $SP(d)$  to its corresponding group that stores all of  $v$ 's non-SP ancestors that belong to the SP DAG  $d$ .

For a given group  $g$ , an element  $e$  in  $g$  has two fields:  $e.node$  and  $e.desc$ . Field  $e.node$  stores the actual non-SP ancestor. Field  $e.desc$  stores the **furthest descendent** of  $e.node$  in  $g$ . We say a node  $v$  is the furthest descendent of  $e.node$  in  $g$  iff (1)  $e.node \leq_d v$ , (2) there exists an element  $x$  in  $g$  such that  $x.node = v$ , (3) for any other element  $y$  in  $g$ , we have  $v \not\leq_d y.node$ . Intuitively, the furthest descendant of  $e.node$  is another non-SP ancestor of  $v$  stored in the same group  $g$  that is a descendant of  $e.node$  and  $e.node$  has no other descendant in the group that is further out.

**Properties of an FOM data structure.** An FOM data structure maintains the following properties.

1. For a non-SP node  $w$  such that  $w \leq v$ , there exists a group containing element  $x$  in  $v.fom$  such that  $x.node = w$ .
2. Given two elements  $x$  and  $y$  in group  $g$ ,  $x.node$  and  $y.node$  are in the same SP DAG  $d$ . If  $x.node <_d y.node$  or  $x.node \parallel_{left}^d y.node$ , then  $x$  is before  $y$  in  $g$ .
3. Given an element  $x$  in group  $g$ , its furthest descendant field is maintained properly. That is, (1)  $x.node \leq_d x.desc$ , (2) there exists an element  $y$  in  $g$  such that  $y.node = x.desc$ , (3) for any other element  $z$  in  $g$ , we have  $x.desc \not\leq_d z.node$ .

Property 1 states that, given a node  $v$  in the NSP DAG,  $v.fom$  has all  $v$ 's non-SP ancestors. Property 2 states that the elements in a group are stored in the English order. Since a group stores only nodes from the same SP DAG, their relationships ( $<_d$  or  $\parallel_{left}^d$ ) can be maintained and queried efficiently using the prior parallel algorithm WSP-Order [50] designed for fork-join parallelism. Property 3 states that the furthest descendants for every element is maintained correctly.

The construction algorithm in F-Order assumes the following helper functions that operate on FOM data structures.

- **FOM-Insert**( $fom, v$ ): Given an instance  $fom$  and a non-SP node  $v$ , FOM-Insert returns a new instance of FOM created by copying over the content of  $fom$  and inserting  $v$  into the appropriate group stored in  $fom$ .
- **FOM-Merge**( $fom_1, fom_2$ ): Given two instances  $fom_1$  and  $fom_2$  of FOM, FOM-Merge returns a new instance of FOM created by merging the contents of  $fom_1$  and  $fom_2$ .

---

### Algorithm 1: F-Order, Construction

---

```

1 Function CommonOrSpawn( $v$ )
2   |  $v.fom = v.parent.fom$ 
3 Function Sync( $v$ )
4   | let  $u$  be the corresponding spawn node of  $v$ 
5   | if  $u.fom \neq v.lparent.fom$  and
6     |  $u.fom \neq v.rparent.fom$  then
7     |   |  $v = \text{FOM-Merge}(v.lparent.fom, v.rparent.fom)$ 
8     |   |  $v.fom = v.lparent.fom$ 
9     |   | else
10    |   |  $v.fom = v.rparent.fom$ 
11 Function CreateOrPut( $v$ )
12 |  $v.fom = \text{FOM-Insert}(v.parent.fom, v)$ 
13 Function Join( $v$ )
14 |  $v.fom = \text{FOM-Merge}(v.local.fom, v.future.fom)$ 

```

---

Algorithm 1 shows the pseudocode of the construction algorithm. F-Order constructs FOMs for all nodes by propagating the presence of non-SP nodes (i.e., create or put) to all its descendants during parallel execution. The algorithm initializes an empty instance of FOM for the source node of the main DAG since it doesn't have any ancestor. Subsequently, it executes nodes of the DAG in any valid

order: a node can be executed when all its ancestors have finished executing. As each node  $v$  executes, the algorithm calls different functions based on  $v$ 's node type.

If  $v$  is a common or spawn node,  $v$  has only one parent  $v.parent$ . Any non-SP ancestor of  $v.parent$  (possibly including  $v.parent$  itself) is also a non-SP ancestor of  $v$ . Thus, the algorithm sets  $v.fom = v.parent.fom$  (line 2).

If  $v$  is a create or put node, besides inheriting all its parent's non-SP ancestors,  $v$  also needs to insert itself into its own FOM data structure. Therefore, the algorithm invokes FOM-Insert to insert itself into  $v.parent.fom$  (line 12), which implicitly creates a new instance of FOM that copies the content from  $v.parent.fom$  and inserts itself into the appropriate group.

If  $v$  is a join node, then  $v$  has two parents: a local parent  $v.local$  and a future parent  $v.future$ . Then the algorithm creates a new instance of FOM by merging  $v.local.fom$  and  $v.future.fom$  (line 14).

Finally, if  $v$  is a sync node, then  $v$  has two parents: a left parent  $v.lparent$  and a right parent  $v.rparent$ . However, it is not always necessary to merge  $v.lparent.fom$  and  $v.rparent.fom$ . By the structural properties of an SP DAG, we know that a sync node  $v$  has a corresponding spawn node  $u$  and two SP sub-DAGs in between: the left sub-DAG  $G_L$  and the right sub-DAG  $G_R$ . The source node of  $G_L$  inherits  $u.fom$ , which can only change when  $G_L$  contains either a create node or a join node (i.e., with an incoming put edge). If  $G_L$  does not contain either create or join node,  $v.lparent.fom$  remains the same as  $u.fom$ . Similarly, the same thing holds for  $G_R$  and  $v.rparent.fom$ . The merge is only necessary when both  $v.lparent.fom$  and  $v.rparent.fom$  have changed compared to  $u.fom$ . Thus, the algorithm checks for whether both have changed, and if so calls FOM-Merge (line 6). Otherwise, if only one changed,  $v.fom$  inherits the one that has changed (lines 8 and 10). If neither has changed, it doesn't matter which one  $v.fom$  inherits from. Take node  $f$  in Figure 1 for instance: its corresponding spawn node is  $c$  and only the right sub-DAG (i.e.,  $e$ ) has its FOM data structure changed from  $c$ . Thus,  $f$  simply inherits its FOM data structure from its right parent  $e$ . The node  $g$ , on the other hand, constructs its FOM data structure by merging the FOM data structures from both of its parents.

For performance reasons, it is important that F-Order calls FOM-Merge only when both sub-DAGs execute nodes that cause their respective FOM data structure to change, a fact that we use when proving the performance bound of F-Order in Section 6.

Lemma 5.1 states that Property 1 always holds for an FOM data structure:

**Lemma 5.1.** *Given a node  $v$ , F-Order constructs a FOM instance  $v.fom$  that stores all the non-SP ancestors (i.e., future create and put nodes) of  $v$  (Property 1).*

**PROOF SKETCH.** One can show this inductively by the nodes executed during parallel execution: provided that the FOM instance(s) of  $v$ 's parent(s) satisfy Property 1, the construction of  $v$ 's FOM also satisfies Property 1.  $\square$

To show that the construction algorithm satisfies the Properties 2 and 3, we need to examine the FOM-Insert and FOM-Merge in more detail. Due to space constraints, we discuss what these functions do at a high-level and omit the full pseudocode.

**Insert Operation.** At FOM-Insert( $fom, v$ ), we create a new FOM  $fom_{new}$  by copying the content of  $fom$  into  $fom_{new}$ . Then we check if  $fom_{new}$  contains a group  $g$  with  $SP(v)$ . If so, we call Group-Insert( $g, v$ ), which returns a new group  $g_{new}$  with a copy of  $g$ 's content and  $v$  added, and we replace  $g$  with  $g_{new}$  in  $fom_{new}$ . If not, we simply create a new empty group  $g_{new}$  with  $v$  added, and add  $g_{new}$  to  $fom_{new}$ .

---

### Algorithm 2: Helper function: Group-Insert

---

```

15 Function Group-Insert( $g, v$ )
16    $g_{new} = \mathbf{new}$  Group() //create a new group
17    $i = j = 1$ 
18   while  $j \leq g.length$  do
19      $x = g[j++]$  //the  $j$ th group element in  $g$ 
20     if  $x.node <_d v \vee x.node \parallel_{left}^d v$  then
21       // copy constructor copying the content of  $x$  into  $y$ 
22        $y = \mathbf{new}$  Group-Element( $x$ )
23       // update furthest descendant if necessary
24       if  $x.desc <_d v$  then  $y.desc = v$ 
25        $g_{new}[i++] = y$  // insert group element  $y$  into  $g_{new}$ 
26     else break // found the right position for  $v$ 
27    $g_{new}[i].node = g_{new}[i].desc = v$ 
28    $i = i + 1$ 
29   //copy the remaining elements of  $g$  into  $g_{new}$ 
30   while  $j \leq g.length$  do
31      $g_{new}[i++] = \mathbf{new}$  Group-Element( $g[j++]$ )
32   return  $g_{new}$ 

```

---

Algorithm 2 shows the helper function Group-Insert, which uses linear search to find the correct position of  $v$  in  $g_{new}$ , which keeps Property 2. Furthermore, for a node  $u$  in  $g_{new}$  such that  $u <_d v$ ,  $v$  checks its furthest descendant to see if it should be replaced with  $v$ ; if so, update it in  $g_{new}$  (line 24). For any  $u'$  that is positioned after  $v$  in  $g_{new}$ ,  $v$  cannot be  $u'$ 's furthest descendant as either  $v <_d u'$  or  $v \parallel_{left}^d u'$  and thus we simply copy over the rest (line 31).

We can conclude the following lemma for FOM-Insert:

**Lemma 5.2.** *Given a FOM  $fom$  that satisfies Properties 2 and 3, FOM-Insert( $fom, v$ ) returns a new FOM with the content of  $fom$  and  $v$  inserted that satisfies Properties 2 and 3.*

**Merge Operation.** FOM-Merge is used in the construction algorithm to merge two FOM instances. At FOM-Merge( $fom_1, fom_2$ ), we create a new FOM  $fom_{new}$ .



We first iterate through groups in  $fom_1$  and insert them into  $fom_{new}$ . We then iterate through groups in  $fom_2$ . For each group  $g_2$  in  $fom_2$ , we check if some group  $g_1$  with  $SP(g_2)$  already exists  $fom_{new}$ . If so, we call `Group-Merge( $g_1, g_2$ )`, which returns a new group  $g$  with merged content, and we replace  $g_1$  with  $g$  in  $fom_{new}$ . If not, we simply insert  $g_2$  into  $fom_{new}$ .

---

**Algorithm 3:** Helper function: Group-Merge
 

---

```

33 Function Group-Merge( $g_1, g_2$ )
34    $g_{new} = \mathbf{new}$  Group() //create a new group
35    $i = j = k = 1$ 
36   while  $i \leq g_1.length \wedge j \leq g_2.length$  do
37      $x = g_1[i]$  //the  $i$ th group element in  $g_1$ 
38      $y = g_2[j]$  //the  $j$ th group element in  $g_2$ 
39     if  $x.node <_d y.node \vee x.node \parallel_{left}^d y.node$  then
40        $z = \mathbf{new}$  Group-Element( $x$ )
41        $i = i + 1$ 
42     else if  $x.node = y.node$  then
43        $z.node = x.node$ 
44       if  $x.desc \leq_d y.desc$  then  $z.desc = y.desc$ 
45       else  $z.desc = x.desc$ 
46        $i = i + 1; j = j + 1$ 
47     else
48        $z = \mathbf{new}$  Group-Element( $y$ )
49        $j = j + 1$ 
50      $g_{new}[k++] = z$  //insert group element  $z$  into  $g_{new}$ 
51 //copy the remaining elements of  $g_1$  or  $g_2$  into  $g_{new}$ 
52 while  $i \leq g_1.length$  do
53    $x = g_1[i++]$ 
54    $g_{new}[k++] = \mathbf{new}$  Group-Element( $x$ )
55 while  $j \leq g_2.length$  do
56    $y = g_2[j++]$ 
57    $g_{new}[k++] = \mathbf{new}$  Group-Element( $y$ )
58 return  $g_{new}$ 

```

---

Algorithm 3 shows `Group-Merge`, which merges two groups while maintaining the English order during merge, and its operation is akin to the merge step in merge sort. Using a similar correctness proof as merge sort, we can conclude the following lemma:

**Lemma 5.3.** *Given  $fom_1$  and  $fom_2$  that satisfy Property 2, `FOM-Merge( $fom_1, fom_2$ )` returns a new FOM instance with the merged content of  $fom_1$  and  $fom_2$  that satisfies Property 2.*

What is not obvious is that Property 3 is also maintained by `Group-Merge`. Consider the process of merging two groups  $g_1$  and  $g_2$ . Given an element  $x$  in  $g_1$ ,  $x.desc$  stores the furthest descendent of  $x.node$  in the scope of  $g_1$ . However, it is possible that there exists a node  $u$  in  $g_2$  such that  $x.desc <_d u$ , which means that  $u$  should become the new  $x.desc$  after merging  $g_1$  and  $g_2$  into  $g_{new}$ . It may seem that `Group-Merge` needs to check  $x.desc$  against every single node in  $g_2$ . It turns out that it is sufficient to only check  $x.desc$  against  $y.desc$

of an element  $y$  in  $g_2$  such that  $x.node = y.node$ , and during the merge we are guaranteed to compare  $x$  against  $y$  for  $x.node = y.node$  (lines 42–46). Lemma 5.4 states that doing so is sufficient to maintain Property 3.

**Lemma 5.4.** *Given two groups  $g_1$  and  $g_2$  that satisfy Property 3, `Group-Merge` merges the content of  $g_1$  and  $g_2$  into  $g_{new}$  while maintaining Property 3 for  $g_{new}$ .*

*Proof.* Given an element  $x$  in  $g_1$ , say there exists a node  $u$  stored in  $g_2$  such that  $u$  is the new furthest descendent of  $x.node$ . Then, we have  $x.node <_d u$ . Also, suppose  $g_2$  is maintained by  $v.fom$ . Then we have  $u \leq v$  since all the nodes stored in  $v.fom$  are  $v$ 's ancestors, which leads to  $x.node < v$ . By Property 1, we know  $v.fom$  stores all of  $v$ 's non-SP ancestors. Thus,  $x.node$  must be stored in some group of  $v.fom$ . Since  $x.node$  and  $u$  are in the same SP DAG, we are guaranteed that  $x.node$  is also in  $g_2$ . Thus, there must exist an element  $y$  in  $g_2$  such that  $y.node = x.node$  and  $y.desc = u$ . Thus, similar to the merge step in merge sort, there must exist an iteration that performs the comparison between  $x$  and  $y$ . As a result, it is sufficient to check for update for  $x.desc$  against  $y.desc$  in such an iteration.  $\square$

## 5.2 Reachability Queries Using FOM

We now describe how we do the reachability query between two nodes  $u$  and  $v$ . Recall the intuitions discussed in Section 4.2. The following lemma formalizes this intuition:

**Lemma 5.5.** *Give two nodes  $u$  and  $v$  in  $G_N$ , we have  $u < v$  iff one of the following is true: 1)  $u <_d v$ ; or 2)  $u \leq_d w$ ,  $w < v$  where  $w$  is a non-SP node (i.e., a create or put node).*

*Proof.* It is clear that if  $u <_d v$ , or  $u <_d w$  and  $w < v$ , we have  $u < v$ . Now we show the other direction also holds. If  $u < v$ , there are two possibilities: 1)  $u \rightsquigarrow_{sp} v$  or 2)  $u \rightsquigarrow_{nsp} v$ . The first case  $u \rightsquigarrow_{sp} v$  is easy to see that  $u$  and  $v$  must be in the same SP DAG and thus we have  $u <_d v$ . Let's consider the second case  $u \rightsquigarrow_{nsp} v$ . Then the path must pass through some create or put node because all outgoing non-SP edges are incident on either create or put nodes. Now we prove that there exists a node  $w$  that  $u \leq_d w$ . If  $u$  is a create or put node, then  $w = u$ . If not,  $u$  must have an outgoing SP edge. Therefore, we can break the non-SP path from  $u$  to  $v$  into a SP path and a non-SP path connected via some create or put node  $w$ . As a result, we have  $u \rightsquigarrow_{sp} w$ , i.e.,  $u <_d w$ .  $\square$

Lemma 5.5 states that in an NSP DAG  $G_N$ , node  $u$  has a path to  $v$  iff: (1)  $u$  and  $v$  are in the same SP DAG  $d$  and there is an SP path between  $u$  and  $v$  in  $d$ ; or (2) the path passes through a create or put node that breaks the path into an SP path and a non-SP path. The first case can be queried efficiently using prior work [50]. The latter case is where we apply the FOM data structures. Specifically, when querying for reachability between  $u$  and  $v$ , F-Order first queries if  $u <_d v$  if they are in the same SP DAG; otherwise, F-Order

searches whether there exists a non-SP ancestor  $w$  stored with  $v.fom$  such that  $u \leq_d w$ .

---

**Algorithm 4:** Group-Search in Reachability Query
 

---

```

59 Function Precedes( $u, v$ )
60   if  $SP(u) = SP(v) \wedge u <_d v$  then return TRUE
61   else
62      $g = v.fom.find(SP(u))$ 
63     if  $g$  then return Group-Search( $u, g$ )
64     else return FALSE
65 Function Group-Search( $u, g$ )
66    $low = 1; high = g.length$ 
67   while  $low \leq high$  do
68      $mid = (low + high)/2; m = g[mid]$ 
69     if  $u \leq_d m.node$  then
70       return TRUE
71     else if  $m.node <_d u \vee m.node \parallel_{left}^d u$  then
72        $low = mid + 1$ 
73     else // must be  $u \parallel_{left}^d m.node$ 
74       if  $u <_d m.desc$  then return TRUE
75       else  $high = mid - 1$ 
76   return FALSE
  
```

---

Algorithm 4 shows the pseudocode for  $Precedes(u, v)$ , which checks if  $SP(u) = SP(v)$  and  $u <_d v$ . If so,  $u < v$  and we are done. If not, we then check if a non-SP path exists using  $v$ 's FOM data structure  $v.fom$ . We search for a group  $g$  with  $SP(u)$  in  $v.fom$ ; if found, we invoke  $Group-Search(u, g)$  to see if a  $w$  exists such that  $u \leq_d w$ . If one is found, then  $u \rightsquigarrow_{nsp} v$ ; if not, we conclude that no path exists between  $u$  and  $v$ .

By Property 2, elements in a group  $g$  is stored in a total English order.  $Group-Search$  uses this fact to apply a process of elimination akin to that in binary search. As hinted before, the process of elimination involves some complications — upon encountering an element  $x$ , in some cases, we must compare  $u$  against  $x.desc$  (line 74) to correctly eliminate half of the remaining elements to check. This leverages Property 3 to guarantee correctness, which we discuss in Lemma 5.6.

**Lemma 5.6.** *Given a node  $u$  and a group  $g$ , the search in  $Group-Search(u, g)$  returns true iff there exists an element  $x$  in  $g$  such that  $u \leq_d x.node$ ; it returns false otherwise.*

*Proof.* First we show that if  $Group-Search(u, g)$  returns true, there exists an element  $x$  in  $g$  such that  $u \leq_d x.node$ . This is evident from the code:  $Group-Search(u, g)$  only returns true when such a node is found (lines 70 and 74).

Now we show the other direction also hold: if an element  $x$  exists in  $g$  such that  $u \leq_d x.node$ ,  $Group-Search(u, g)$  returns true. That is, if such  $x$  exists,  $Group-Search(u, g)$  will find it by correctly eliminating half of the remaining elements that we don't need. We will examine this by cases on the if conditions executed in  $Group-Search(u, g)$ .

By Property 2, if  $x.node <_d y.node$  or  $x.node \parallel_{left}^d y.node$ , then  $x$  is positioned before  $y$  in  $g$ . Let's suppose

that the element we are looking for is  $x$  and it exists. If  $g[mid].node <_d u$  (first part of condition in line 71), then obviously  $g[mid].node <_d x.node$  and we need to only search the array elements positioned after  $g[mid]$ . The code correctly performs the elimination (line 72).

Now we show if  $g[mid].node \parallel_{left}^d u$  (second part of condition in line 71), then we have either  $g[mid].node <_d x.node$  or  $g[mid].node \parallel_{left}^d x.node$ . Consider the left sub-DAG  $G_L$  containing  $g[mid].node$  and the corresponding right sub-DAG  $G_R$  containing  $u$ . Say  $G$  is the SP sub-DAG consists of  $G_R$  and  $G_L$ . Then there are two possibilities for where  $x.node$  can be: either  $x.node$  is in  $G_R$ , then  $g[mid].node \parallel_{left}^d x.node$ , or  $sink(G)$  (the sink node of  $G$ )  $\leq_d x.node$ , then  $g[mid].node <_d x.node$ . In either case,  $g[mid]$  must be positioned before  $x$  in  $g$ , and the code correctly performs the elimination (line 72).

We now consider the case that  $u \parallel_{left}^d g[mid].node$  (line 73). Again, consider the SP sub-DAG  $G$  with the left and right sub-DAGs  $G_L$  and  $G_R$ . Say  $u$  is in  $G_L$  and  $g[mid].node$  is in  $G_R$ . Then it could be either  $sink(G) \leq_d x.node$  or  $x.node \in G_L$ . In the first case,  $x.node$  is also a descendent of  $g[mid].node$ . Recall that by Property 3,  $g[mid].desc$  stores the furthest descendent node of  $g[mid].node$ . If  $g[mid].node <_d x.node$ , we are guaranteed that  $sink(G) \leq_d g[mid].desc$ . Otherwise, we have  $g[mid].desc <_d sink(G)$  and as a result  $g[mid].desc <_d x.node$ , which contradicts that  $g[mid].desc$  is  $g[mid].node$ 's furthest descendent. Thus, if  $sink(G) \leq_d x.node$  is true, we must have  $sink(G) \leq_d g[mid].desc$ , which leads to  $u <_d g[mid].desc$  (line 74). Now we consider the second case,  $x.node \in G_L$ . In this case, we have obviously  $x.node \parallel_{left}^d g[mid].node$ . By Property 2, we can conclude that target  $x$  must be between positions  $low$  and  $mid - 1$ , and the code correctly performs the elimination (line 75).  $\square$

The last part of the proof in Lemma 5.6 makes it clear why we must store the furthest descendent with every element — even if the target node is in the part of the array that we eliminate, we are guaranteed to find it as another element's furthest descendent field. Take nodes  $i$  and  $f$  in Figure 1 for instance. Say  $f$  executes second, and we want to check if  $f$  is reachable from  $i$ . We find the group  $g$  with  $SP(i) = B$  and invoke  $Group-Search(i, g)$ . Even though  $Group-Search$  eliminates the second half of  $g$ , it still concludes that  $f$  is reachable from  $i$  (i.e., returns true), as we have  $n$  stored as  $l.desc$  and  $i <_d n$ .

By Lemmas 5.5 and 5.6 and by the operations of  $Precedes$ , we can show the following theorem.

**Lemma 5.7.** *Provided that  $v.fom$  satisfies Properties 1–3,  $Precedes(u, v)$  correctly returns true iff  $u \rightsquigarrow v$  in  $G_N$ .*

**Theorem 5.8.** *Given two executed nodes  $u$  and  $v$  in NSP DAG  $G_N$ . F-Order correctly answers the reachability query between  $u$  and  $v$ .*

*Proof.* By Lemmas 5.1, 5.2, 5.3, and 5.4, one can inductively show that Properties 1–3 on any FOM is maintained at all times. Then by Lemma 5.7, F-Order can answer the reachability query correctly provided that the FOM instance of each executed node satisfies Properties 1–3.  $\square$

## 6 The Performance Bound of F-Order

This section proves the performance bound of F-Order. To perform a reachability query between two nodes in the same SP DAG, we utilize the parallel algorithm WSP-Order from Utterback et al. [50], including scheduling support for maintaining concurrent order-maintenance (OM) data structures. In our work, we augmented our scheduler similarly to provide support for such concurrent OM data structures.

To bound the overhead of WSP-Order and the use of concurrent OM data structures, we invoke the following lemma shown by Utterback et al. [50]:

**Lemma 6.1.** *Given an SP DAG with work  $T_1$  and span  $T_\infty$ , one can perform reachability maintenance and queries on the SP DAG in time  $O(T_1/P + T_\infty)$  on  $P$  processors.*

To complete the time bound for F-Order, we need to account for the additional overhead incurred by the maintenance of and queries on the FOM data structures.

**Lemma 6.2.** *Given an NSP DAG with  $k$  number of future operations, the total number of FOM-Insert invocations is at most  $O(k)$ , each with  $O(k)$  work.*

*Proof.* Each FOM data structure stores only non-SP ancestors, i.e., ancestors that are either create or put nodes. Since there are  $k$  future operations, each FOM instance can have at most  $O(k)$  elements. Therefore, it takes  $O(k)$  time to perform a single FOM-Insert operation (which creates a new copy so linear work is required). Moreover, by Algorithm 1, F-Order performs FOM-Insert only on create and put nodes, and thus FOM-Insert is invoked at most  $O(k)$  times.  $\square$

**Lemma 6.3.** *Given an NSP DAG with  $k$  number of future operations, the total number of FOM-Merge invocations is at most  $O(k)$ , each with  $O(k)$  work.*

*Proof.* FOM-Merge operates on inputs of size at most  $O(k)$ , and thus each operation incurs at most  $O(k)$  work (like the merge operation in merge sort). By Algorithm 1, F-Order performs FOM-Merge only on join and sync nodes. Since there are at most  $k$  future operations, there are at most  $k$  join nodes. What's not as obvious is bounding the number of FOM-Merge invocations due to sync nodes.

By Algorithm 1, F-Order performs FOM-Insert on a sync node only when the FOM data structures from both of its parents have changed from that of its corresponding spawn node.

Consider an SP DAG constructed recursively using  $n$  parallel compositions. We will call the outer-most SP DAG a level-0 DAG, or  $G^0$ , and call its left and right sub-DAGs level-1 DAGs,  $G_L^0$  and  $G_R^0$ , or simply  $G^1$ . Since this DAG is

constructed with  $n$  parallel compositions, there are  $n$  levels of nested SP DAGs with  $n$  sync nodes, one for each level. Without loss of generality, we will show that, by adding a new incoming or outgoing non-SP edge into this DAG, the addition incurs at most one extra FOM-Merge on the closest enclosing sync node, but not on the other sync nodes at the outer level.

Imagine today we add an outgoing non-SP edge to the left sub-DAG at level  $i$ ,  $G_L^i$ . Consider both the sync node  $s$  that joins together  $G_L^i$  and  $G_R^i$  and its corresponding spawn node  $f$ . The FOM instance from  $s$ 's left parent would change. This may or may not prompt a FOM-Merge at  $s$ .

**Case 1:** Let's consider the case where it did. Then, it must be that there is also an incoming or outgoing non-SP edge in  $G_R^i$ , causing the FOM instance from  $s$ ' right parent to change from that of the  $f$ . *fom*. If so, one extra FOM-Merge would be incurred compared to not adding that non-SP edge. From the perspective of the sync node  $s'$  at level  $i-1$ , however, this change does not affect whether  $s'$  performs a FOM-Merge or not. Without loss of generality, say the DAG consist of  $G_L^i$  and  $G_R^i$  is the left sub DAG at level  $i-1$  (i.e.,  $G_L^{i-1}$ ). Without adding a new non-SP edge to  $G_R^{i-1}$ ,  $s'$  will simply inherit the results of FOM-Merge at  $s$ .

**Case 2:** Let's consider the other case where this addition did not prompt  $s$  at level  $i$  to perform FOM-Merge. Then, it must be that, the FOM from  $s$ 's right parent is the same as  $f$ . *fom*, the FOM from the corresponding spawn node. Then,  $s$  would have simply inherit the FOM from  $G_L^i$ , incurring zero additional FOM-Merge operations at level  $i$ . Now, it may incur an extra FOM-Merge at the sync node at level  $j$  for some  $j < i$ . But the same argument from case 1 can be applied to level  $j$  and the FOM-Merge stops at level  $j$  and not further.

Thus, we can conclude that there are at most  $O(k)$  total FOM-Merge operations, each with  $O(k)$  work.  $\square$

Now we put the overhead due to maintaining FOM data structures together.

**Lemma 6.4.** *Given an NSP DAG with work  $T_1$  and span  $T_\infty$ . F-Order runs in time  $O((T_1 + k^2)/P + T_\infty k)$  on  $P$  processors to construct the reachability data structure, where  $k$  is the number of future operations.*

*Proof.* By Lemmas 6.2 and 6.3, the construction of FOM data structures incurs at most  $O(k^2)$  work and in the worst case,  $O(k)$  multiplicative overhead on the span (if F-Order performs a FOM-Merge or FOM-Insert on every single node along the span). Adding these overheads and applying Lemma 6.1, we obtain the bound.  $\square$

The bound shown in Lemma 6.4 accounts for only the construction overhead. To show the full performance bound, we must also account for the query overhead.

**Lemma 6.5.** *Given two nodes  $u$  and  $v$  in an NSP DAG  $G_N$ , a single reachability query  $\text{Precedes}(u, v)$  runs in time  $O(\lg k)$ ,*

where  $\hat{k}$  is the number of non-SP ancestors of  $v$  from the SP DAG containing  $u$ .

*Proof.* In the worst case, there is no direct SP path between  $u$  and  $v$ , and F-Order needs to perform a search to check if  $v.fom$  stores any descendent node of  $u$ . Identifying a group  $g$  with  $SP(u)$  in  $v.fom$  takes constant time; if  $g$  exists, invoking Group-Search( $u, g$ ) takes at most  $O(\lg \hat{k})$  time (akin to binary search).  $\square$

**Theorem 6.6.** *Given an NSP DAG  $G_N$  with work  $T_1$  and span  $T_\infty$ , F-Order can race detect  $G_N$  in parallel in time  $O((T_1 \lg \hat{k} + k^2)/P + T_\infty(k + \lg r \lg \hat{k}))$  on  $P$  processors, where  $k$  is the number of future operations,  $r$  is the maximum number of readers for a single memory location, and  $\hat{k}$  is the maximum number of non-SP nodes in the same SP DAG.*

*Proof.* The total overhead incurred due to reachability queries is related to how the access history is managed. As discussed in Section 4, the number of readers per memory location at a given moment can be large. However, one can still show that, the total number of reachability queries throughout the execution is bounded by  $2\times$  the number of reads during execution [1]. Each query itself incurs  $O(\lg \hat{k})$  overhead Lemma 6.5. Thus, given an NSP DAG with work  $T_1$  and span  $T_\infty$ , the total work incurred by reachability queries using F-Order is  $O(T_1 \lg \hat{k})$ .

Now we consider how queries impact the span of race detection. Given a single node  $u$  along the span of  $G_N$  that writes to memory location  $l$ . Since F-Order needs to perform reachability queries between  $u$  and every single reader in  $reader\_list(l)$ , assuming are at most  $r$  readers in  $reader\_list(l)$ , the total number of queries performed when executing  $u$  is at most  $r$ . Since all the queries can be computed independently of each other, such the overall query overhead has a span of  $O(\lg r \lg \hat{k})$ . In the worst case, every node  $u$  along the span of  $G_N$  incurs such query overhead. Then, combining Lemma 6.4, the bound follows.  $\square$

## 7 Implementation and Empirical Analysis

This section briefly describe our prototype implementation of F-Order and empirically evaluates its performance. We evaluate F-Order’s performance across six different benchmarks and compare it to FutureRD,<sup>4</sup> a state-of-the-art sequential race detector for futures [51]. FutureRD provides the best sequential running time for race detecting the structured use of futures which imposes certain programming restrictions on where the future get can occur. For race detecting general futures, the algorithm by Utterback et al. [51] has an additional  $\alpha(m, n)$  overhead compared to the algorithm by Agrawal et al. [1], but the algorithm by Agrawal et al. [1] has never been implemented.

<sup>4</sup>The codebase of FutureRD can be obtained at <https://github.com/wustl-pctg/futurerd>.

bench	$N$	$B$	reads	writes	futures	strands	avg
sw-sf	2048	64	$8.59 \times 10^9$	$4.20 \times 10^6$	1024	2054	1.0
hw-sf	10 (images)	-	$1.73 \times 10^{10}$	$1.64 \times 10^8$	3672	9914	14.05
sort	$10^7$	8192	$2.75 \times 10^8$	$2.22 \times 10^8$	14463	60030	2.95
mm	2048	64	$1.72 \times 10^{10}$	$1.43 \times 10^8$	18724	79577	20.94
smm	2048	64	$9.40 \times 10^8$	$2.50 \times 10^5$	16387	70822	8.24
ferret	simlarge	-	$5.40 \times 10^9$	$6.23 \times 10^8$	256	1280	13.09
sw-gf	2048	64	$8.59 \times 10^9$	$4.20 \times 10^6$	1024	5124	1.0
hw-gf	10 (images)	-	$1.73 \times 10^{10}$	$1.64 \times 10^8$	4590	11750	13.1

**Figure 2.** The characteristics of the benchmarks. The sw and hw benchmarks have two implementations: structured(sf) and general futures(gf).

Empirical results indicate that, even though our parallel race detector incurs higher overhead on one-core execution, the overhead is never more than  $3\times$  compared to FutureRD (in fact much less for all benchmarks except for hw). Thus, the fact that we can race detect while executing the program in parallel quickly pays off in absolute execution times.

**Implementation.** We have implemented F-Order by extending the Cilk-F runtime system [43], a work-stealing runtime system that supports the use of futures. In the implementation of F-Order, we employed WSP-Order [50] for maintaining and querying the reachability between two nodes that are in the same SP DAG (i.e.,  $<_d$  and  $\parallel_{left}^d$ ). We implemented the augmentation necessary in the Cilk-F runtime as proposed by Utterback et al. [50]. As discussed in Section 6, such an augmentation is necessary in order to provide the desired performance bound. The construction functions of the FOM are called via instrumentation inserted into the parallel control constructs of Cilk-F. The instrumentation on memory accesses are inserted via ThreadSanitizer [42] pass implemented in LLVM.

**Benchmarks.** Six benchmarks are used: matrix multiplication (mm), the Strassen matrix multiplication algorithm (smm), parallel merge sort (sort), the Heart Wall application (hw) from the Rodinia suite [11], Smith-Waterman for sequence alignment (sw), and a content-based image similarity search modified from the PARSEC benchmark suite (ferret) [6]. We have implemented all the benchmarks with structured use of futures. The sw and hw benchmarks, in addition, have a second implementation with a general use of futures that imposes no restrictions.

FutureRD distinguishes between structured and general use of futures, and provides better running time for structured use of futures. Our race detector does not distinguish between structured versus general use of futures and provides the same bound for both uses.

The characteristics of these benchmarks are shown in Figure 2, including the input sizes and serial base case sizes used. We also measured the average group sizes; the measurement indicates that average group sizes (related to  $\hat{k}$ ) are indeed small across benchmarks.

**Experimental Setup.** All experiments were run on a machine with two 20-core Intel Xeon Gold 6148 processors,

<i>bench</i>	<i>configuration</i>	$T_1$	$T_{20}$	<i>FutureRD</i>
sw (structured)	baseline	21.88	2.26 [9.67×]	21.57
	reachability	22.49 (1.02×)	2.29 [9.79×]	21.52 (0.99×)
	full	697.06 (31.85×)	96.96 [7.18×]	562.55 (26.08×)
hw (structured)	baseline	15.41	0.98 [15.60×]	15.5
	reachability	17.95 (1.16×)	1.03 [17.32×]	15.5 (1.0×)
	full	943.33 (61.18×)	72.12 [13.07×]	381.85 (24.63×)
sort (structured)	baseline	1.33	0.07 [17.17×]	1.34
	reachability	5.05 (3.77×)	0.38 [13.04×]	1.34 (1.0×)
	full	30.62 (22.87×)	2.2 [13.92×]	19.48 (14.48×)
mm (structured)	baseline	8.48	0.43 [19.30×]	8.47
	reachability	12.97 (1.52×)	0.68 [19.05×]	8.47 (1.0×)
	full	484.48 (57.13×)	24.44 [19.81×]	320.46 (37.86×)
smm (structured)	baseline	2.42	0.14 [16.60×]	-
	reachability	2.84 (1.17×)	0.16 [17.30×]	-
	full	51.32 (21.14×)	2.69 [19.01×]	-
ferret (structured)	baseline	7.45	0.65 [11.41×]	7.33
	reachability	7.64 (1.02×)	0.66 [11.54×]	7.28 (0.99×)
	full	337.23 (45.26×)	32.76 [10.29×]	298.18 (40.68×)
sw (general)	baseline	21.79	2.28 [9.54×]	21.64
	reachability	24.93 (1.14×)	2.28 [10.91×]	21.58 (0.99×)
	full	704.72 (32.32×)	100.1 [7.04×]	610.75 (28.22×)
heartwall (general)	baseline	15.42	0.99 [15.48×]	15.5
	reachability	18.56 (1.2×)	1.08 [17.09×]	15.5 (1.0×)
	full	934.47 (60.6×)	68.13 [13.71×]	488.13 (31.46×)

**Figure 3.** Performance of the benchmarks with F-Order and FutureRD for race detection. Execution time on  $P$  processors,  $T_P$ , is given in seconds. Numbers in the parentheses show the overhead compared to the baseline. Numbers in the brackets show the scalability relative to  $T_1$  of the same configuration. Measurements of *smm* running with FutureRD is not available because it segfaulted.

clocked at 2.40 GHz, with hyperthreading disabled. Each core has separate private 32 KB L1 data and 32 KB L1 instruction caches, and a 1 MB private L2 cache. Each socket has a 27.5 MB shared L3 cache. The machine has 768 GB of main memory. We limit execution to the first 20 cores, located on the first socket of the machine, in order to avoid NUMA overhead. All software is compiled with LLVM/Clang 3.4.1 with -O2 optimizations running on Linux kernel version 4.15. Each data point is the average of 3 runs.

For each benchmark, we ran three different configurations: **baseline**, where the benchmark is compiled without any race detection enabled; **reachability**, where the benchmark is compiled with only the reachability component but not the access history; and **full**, where the benchmark is compiled with full race detection.

### Practical Performance of F-Order

Figure 3 shows our measurements for the benchmarks. With the exception of the *sort* benchmark, we see that the reachability versions of F-Order incur very little overhead when compared to the baseline versions. The overhead is more pronounced in *sort* because a majority of the futures

created do little more than generate more futures,<sup>5</sup> and even in the serial base case the work is only  $\theta(B \lg B)$ , where  $B$  is the problem size of the serial base case. Moreover, we expect the reachability overhead of FutureRD is less than that of F-Order on benchmarks using structured futures because its reachability algorithm designed for structured futures is more efficient than its algorithm designed for general futures. F-Order, however, cannot take advantage of the restrictions imposed by structured use of futures.

Full race detection versions incur a large increase in overhead in both F-Order and FutureRD, which comes from the combination of the memory instrumentation and the sheer quantity of reachability queries. The additional overhead of full race detection in F-Order compared to FutureRD is the price one pays to enable parallel race detection. In F-Order, each query incurs  $O(\lg \hat{k})$  instead of constant time (which is the case for FutureRD). This overhead is the most evident in *hw*; this is because the structure of the parallelism and the memory access pattern cause significantly more non-SP queries than in any of our other benchmarks. Even in the case of the high overhead *hw*, however, the full race detection version of the benchmarks maintain scalability comparable to that of the baselines. The higher overhead of non-SP queries in F-Order can be offset by the scalability that F-Order gains. As shown in Figure 3, the absolute running times of F-Order on 20 cores are significantly faster than the running time of FutureRD. In our evaluation, the running times of all benchmarks with F-Order on 4 cores or more can beat the running time of FutureRD.

## 8 Conclusion

In this paper, we propose a parallel race detection algorithm for task parallel programs that employ futures. In the literature, researchers have distinguished use of futures to be structured versus general in the context of studying performance bounds [18, 43]. Moreover, Utterback et al. [51] showed that, when race detecting sequentially at least, one can exploit the structured use of futures to obtain lower race detection overhead. Unfortunately, their algorithms cannot be parallelized easily. Our proposed algorithm does not currently exploit the structured use of futures to gain a performance advantage. As an interesting research direction, we plan to investigate whether it's possible to obtain a more efficient parallel race detection algorithm when one restricts to only structured use of futures.

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<sup>5</sup>This is also true for *mm* and *smm*; their serial base cases, however, perform much more work,  $\theta(B^3)$ .

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