CSE341T/CSE549T **August 25, 2014**

Commonly Used Formulae

Series

1. $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

2. Sum of squares
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Sum of cubes
$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

4. Geometric series
$$\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$$

5. Geometric series with $|x| < 1$
$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

6. Harmonic numbers
$$H_n = \sum_{i=1}^{n} \frac{1}{i} = \ln n + \Theta(1)$$

Probability

More background on probability in Appendix C.2 of the textbook.

1. Definition of expectation.
$$E[X] = \sum x \Pr\{X = x\}$$

2. Linearity of Expectation
$$E[X_1 + X_2] \leq E[X_1] + E[X_2]$$

3. If X_1 and X_2 are independant,
$$E[X_1 X_2] \leq E[X_1]E[X_2]$$

4. Markov's Inequality
$$\Pr\{X \geq a\} \leq \frac{E[X]}{a}$$
5. If X is an indicator random variable
$$E[X] = \Pr X = 1$$

Other Useful Inequalities

1. Stirling's approximation
$$n! \leq \left(\frac{n}{e}\right)^n$$
2. $\binom{n}{k}$ notation (called n choose k): Given a set of n distinct elements, $\binom{n}{k}$ represents the number of distinct k element subsets of this set. In other words, it is the number of ways of choosing k elements from a given set of n distinct elements. If n and k are non-negative and $k \leq n$, then
$$\binom{n}{k} \leq \frac{n!}{k!(n-k)!}$$
3. $$\binom{n}{k} \leq \frac{n^k}{k!}$$
4. $$\binom{n}{k} \leq \left(\frac{ne}{k}\right)^k$$
5. $$\binom{n}{k} \geq \left(\frac{n}{k}\right)^k$$
6. $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$
7. $$\ln(1+x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \ldots$$
8. $e^t \geq 1 + t$
9. For positive t, n
$$\left(1 + \frac{t}{n}\right)^n \leq e^t \leq \left(1 + \frac{t}{n}\right)^{n+t/2}$$
The above inequality also holds for negative t if $n \geq 1$ and $|t| \leq n$.

Asymptotic Notation

For any $\varepsilon > 0$ and any k, $n^\varepsilon = \Omega(\lg^k n)$.