CPU Utilization Control in Distributed Real-Time Systems

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IM 2009: Recent Advances in the Application of Control Theory to Network and Service Management

Highlight

- Common class of computing problems
  - MIMO: multi-input (knobs), multi-output (objectives)
  - Coupling between objectives.
  - Constraints on knobs.

- Model Predictive Control
  - Optimization + Prediction + Feedback

Why CPU Utilization Control?

- Overload protection
  - CPU over-utilization → system crash
- Meet response time requirement
  - CPU utilization < bound → meet deadlines

Challenge: Uncertainties

- Execution times?
  - Unknown sensor data or user input
- Request arrival rate?
  - Aperiodic events
  - Bursty service requests
- Disturbance?
  - Denial of Service attacks

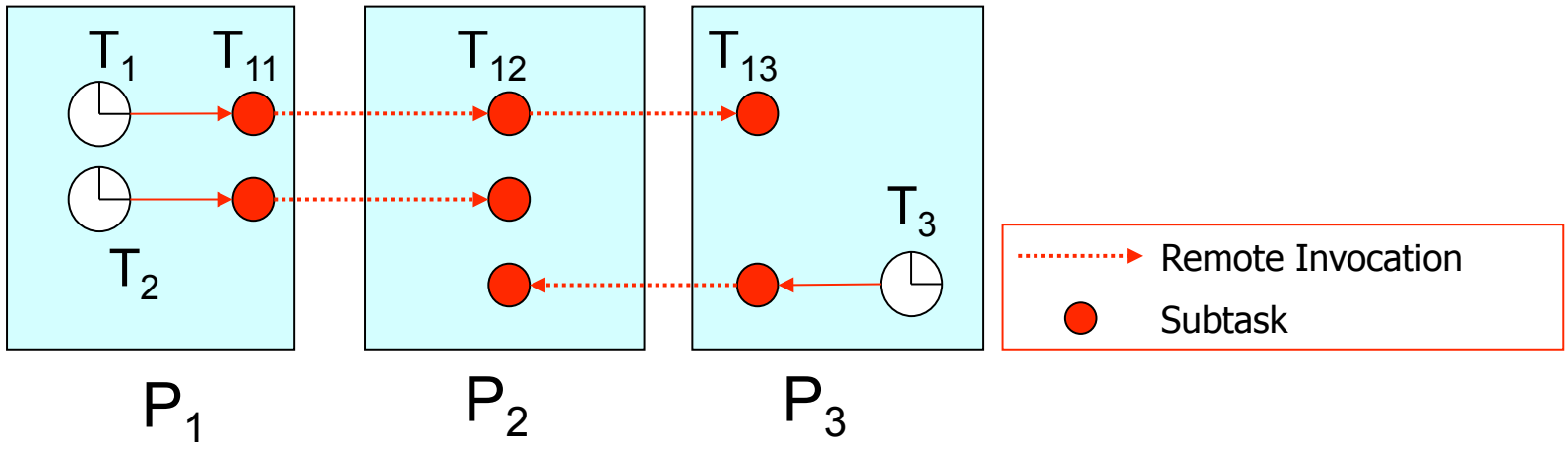
Control-theoretic approach

→ Robust utilization control in face of workload uncertainty

End-to-End Tasks

Distributed Real-Time Systems

- Periodic task T_i = sequence of subtasks $\{T_{ij}\}$ on different processors
  - All the subtasks of a task run at a same rate
- Task rate can be adjusted
  - Within a range
  - Higher rate → higher utility



Problem Formulation

- B_i: Utilization set point of processor P_i $(1 \le i \le n)$
- $u_i(k)$: Utilization of P_i in the k^{th} sampling period
- $r_j(k)$: Rate of task T_j $(1 \le j \le m)$ in the k^{th} sampling period

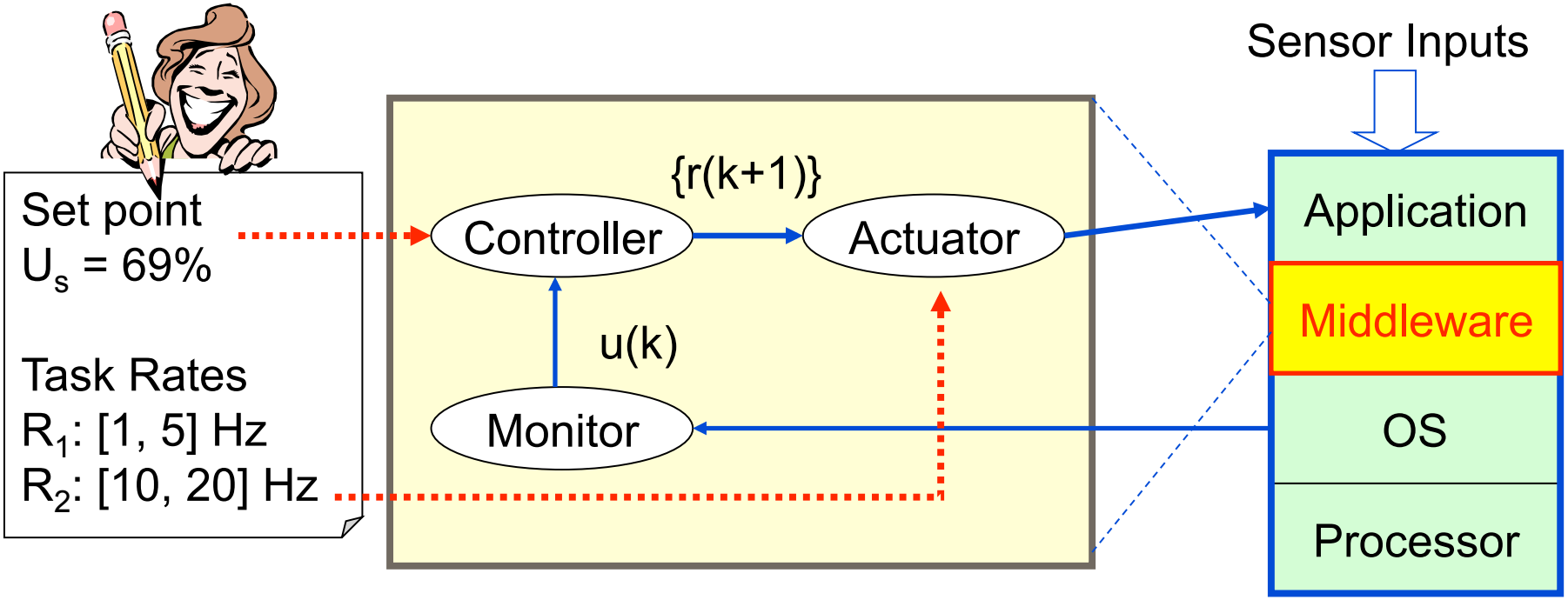
$$\min_{\{r_j(k) | 1 \le j \le n\}} \sum_{i=1}^{n} (B_i - u_i(k))^2$$

subject to rate constraint:

$R_{min,j} \le r_j(k) \le R_{max,j}$ $(1 \le j \le m)$

Single-Input-Single-Output (SISO) Control

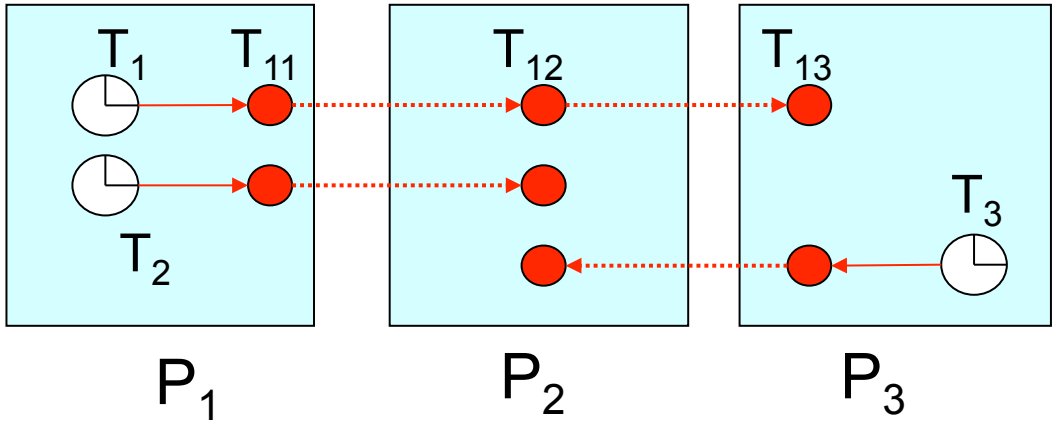
Single Processor



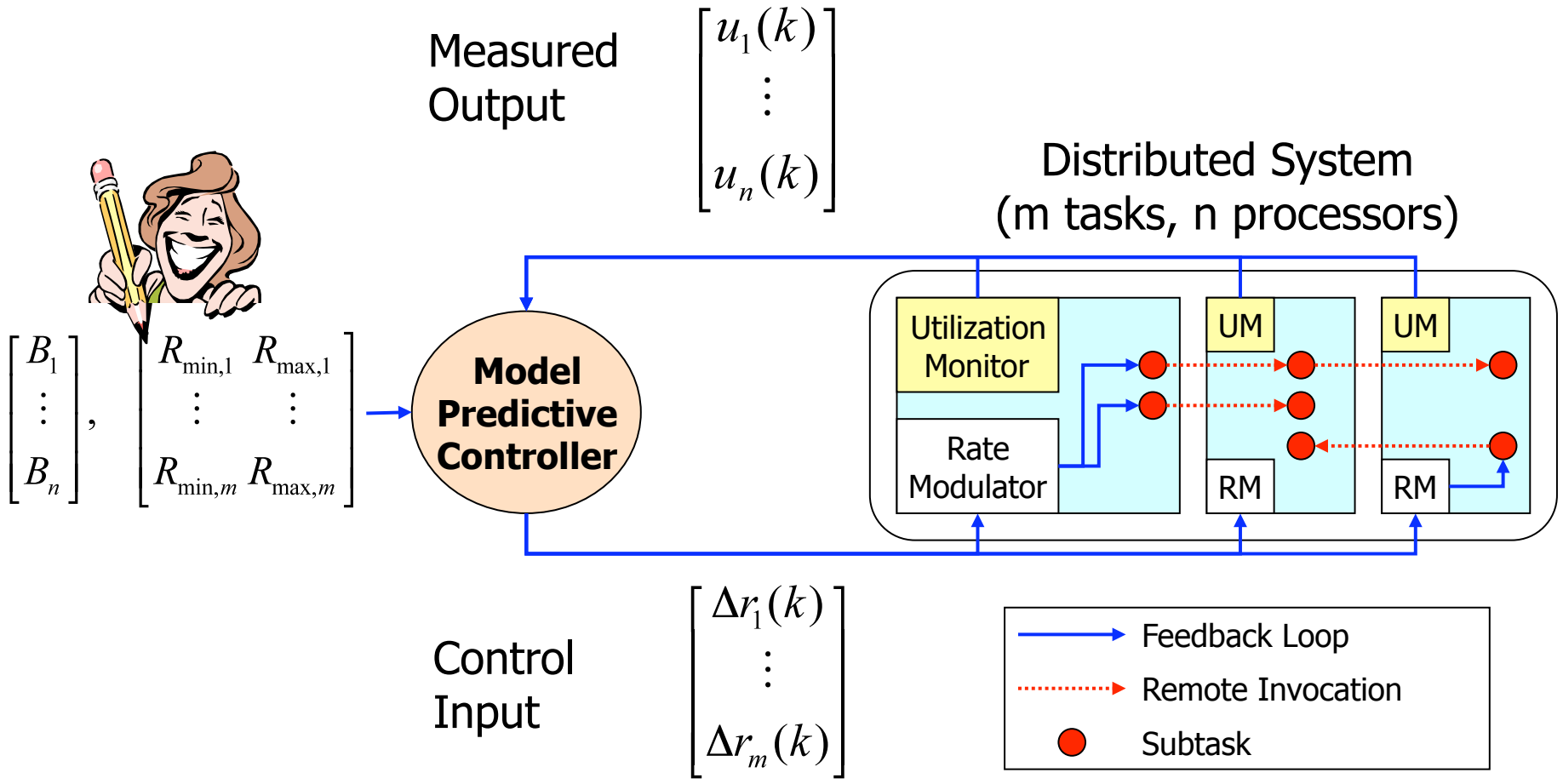
C. Lu, X. Wang, and C. Gill, Feedback Control Real-Time Scheduling in ORB Middleware, IEEE Real-Time and Embedded Technology and Applications Symposium (RTAS'03), May 2003.

New in Distributed Systems

- Need to control utilization of multiple processors
- Utilization of different processors are coupled with each other due to end-to-end tasks
  - → Replicating a SISO controller on all processors does not work!
- Constraints on task rates



EUCON: Multi-Input-Multi-Output Control



C. Lu, X. Wang and X. Koutsoukos, Feedback Utilization Control in Distributed Real-Time Systems with End-to-End Tasks, IEEE Transactions on Parallel and Distributed Systems, 16(6): 550-561, June 2005.

Control Theoretic Methodology

1. Derive a dynamic model of the system
2. Design a controller
3. Analyze stability

Dynamic Model: One Processor

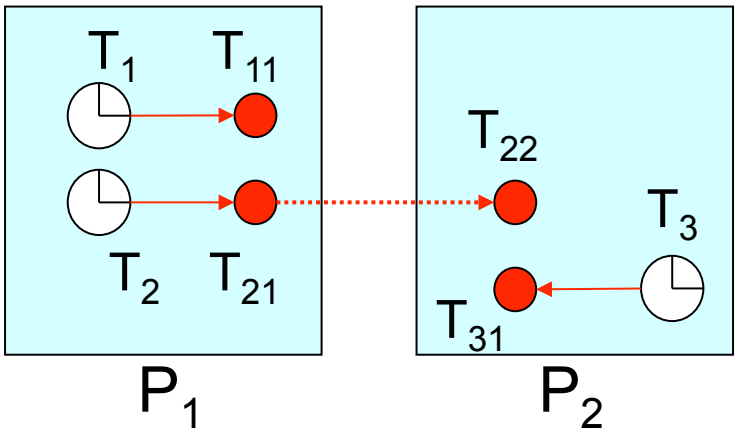
$$u_i(k) = u_i(k-1) + g_i \sum_{T_{jl} \in S_i} c_{jl} \Delta r_j(k-1)$$

- S_i: set of subtasks on P_i
- c_{jl}: estimated execution time of T_{il}
- g_i: utilization gain of P_i
  - ratio between actual and estimated change in utilization
  - models **uncertainty** in execution times

Dynamic Model: Multiple Processors

$$\boldsymbol{u}(k) = \boldsymbol{u}(k\text{-}1) + \boldsymbol{GF}\Delta\boldsymbol{r}(k\text{-}1)$$

- **G:** diagonal matrix of utilization gains
- **F**: subtask allocation matrix
  - models the coupling among processors
  - $f_{ij} = c_{jl}$ if task T_j has a subtask T_{jl} on processor P_i
  - $f_{ij} = 0$ if T_j has no subtask on P_i



$$F = \begin{bmatrix} c_{11} & c_{21} & 0 \\ 0 & c_{22} & c_{31} \end{bmatrix}$$

Model Predictive Control

- Suitable for coupled MIMO control problems with constraints.
- Compute input to minimize cost over a future interval.
  - Cost function: tracking error and control cost.
  - Predict cost based on a system model and feedback.
  - Compute input subject to constraints.
- Optimization + Prediction + Feedback

Cost Function

- Cost

$$V(k) = \sum_{i=1}^{P} \left\| u(k+i) - ref(k+i) \right\|^2 + \sum_{i=0}^{M-1} \left\| \Delta r(k+i) - \Delta r(k+i-1) \right\|^2$$

Tracking Error — Control Cost

- Reference trajectory: exponential convergence to **B**

$$ref(k+i) = B - e^{-\frac{T_s}{T_{ref}} i} (B - u(k))$$

Model Predictive Controller

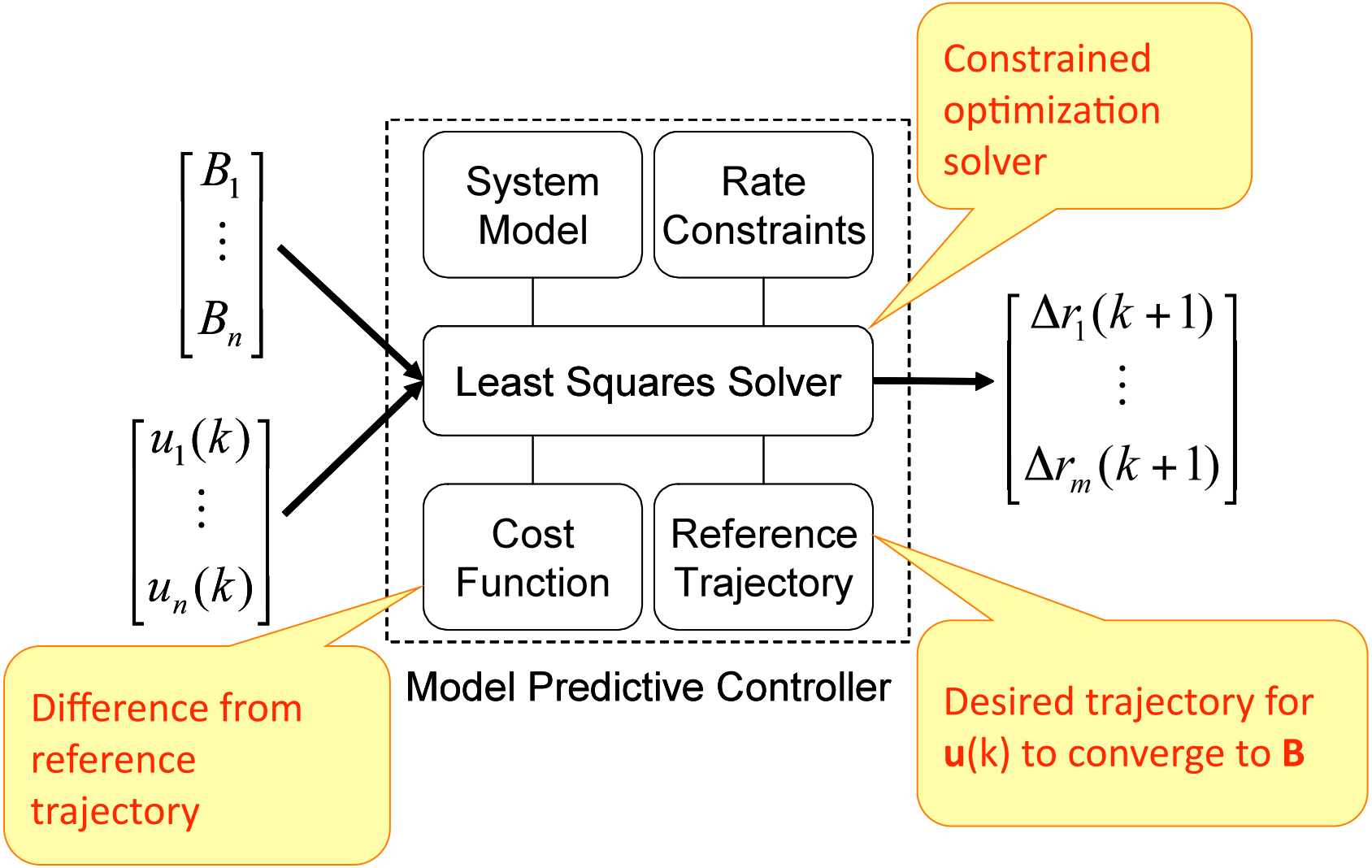
At the end of each sampling period

- Compute inputs in future sampling periods
  $\Delta \mathbf{r}(k), \Delta \mathbf{r}(k+1), \ldots \Delta \mathbf{r}(k+M-1)$
  to minimize the cost function
- Cost is predicted using
  (1) feedback $u(k-1)$
  (2) approximate dynamic model
- Apply $\Delta \mathbf{r}(k)$ to the system

At the end of the next sampling period

- Shift time window and re-compute $\Delta \mathbf{r}(k+1), \Delta \mathbf{r}(k+2), \ldots \Delta \mathbf{r}(k+M)$ based on feedback

EUCON Controller

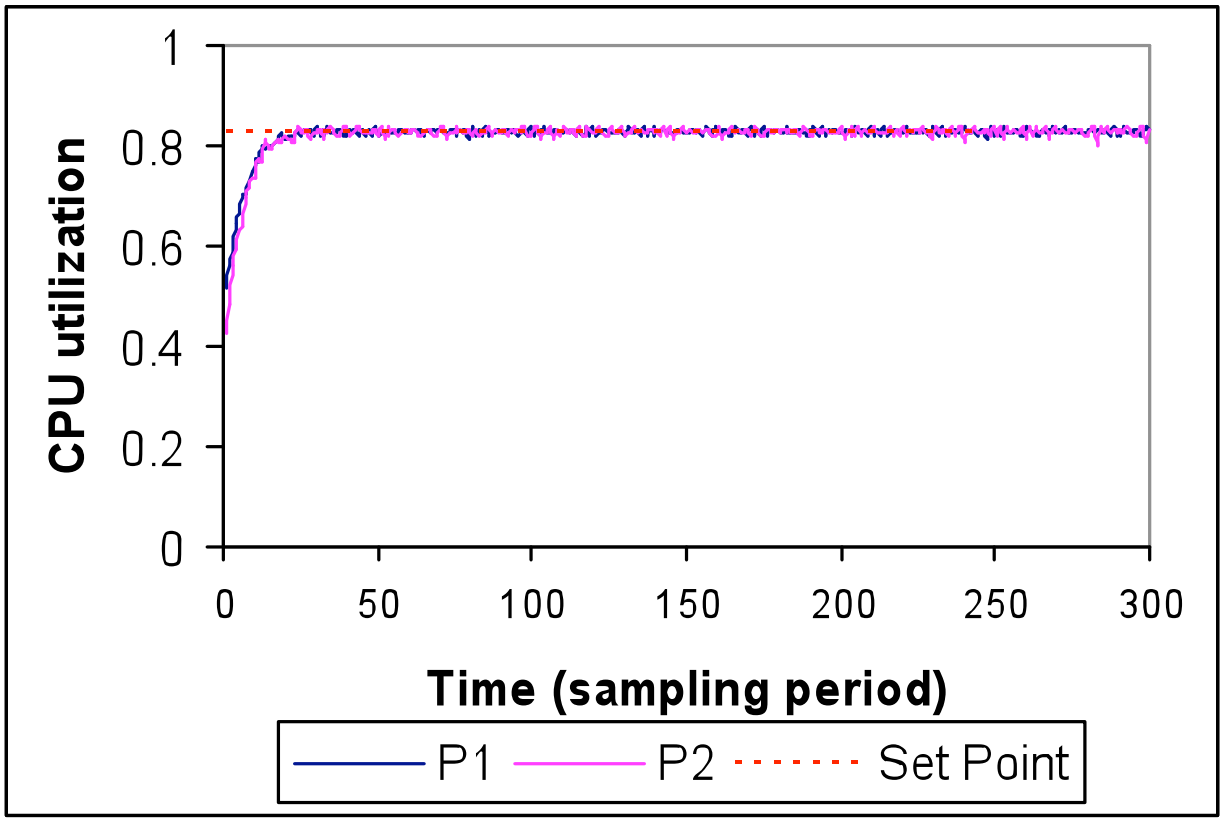


Stability Analysis

- Stability: utilization of all processors converge to set points
- Derive stability condition → range of **G**
  - Tolerable variation of execution times

→ Provides analytical assurance despite uncertainty

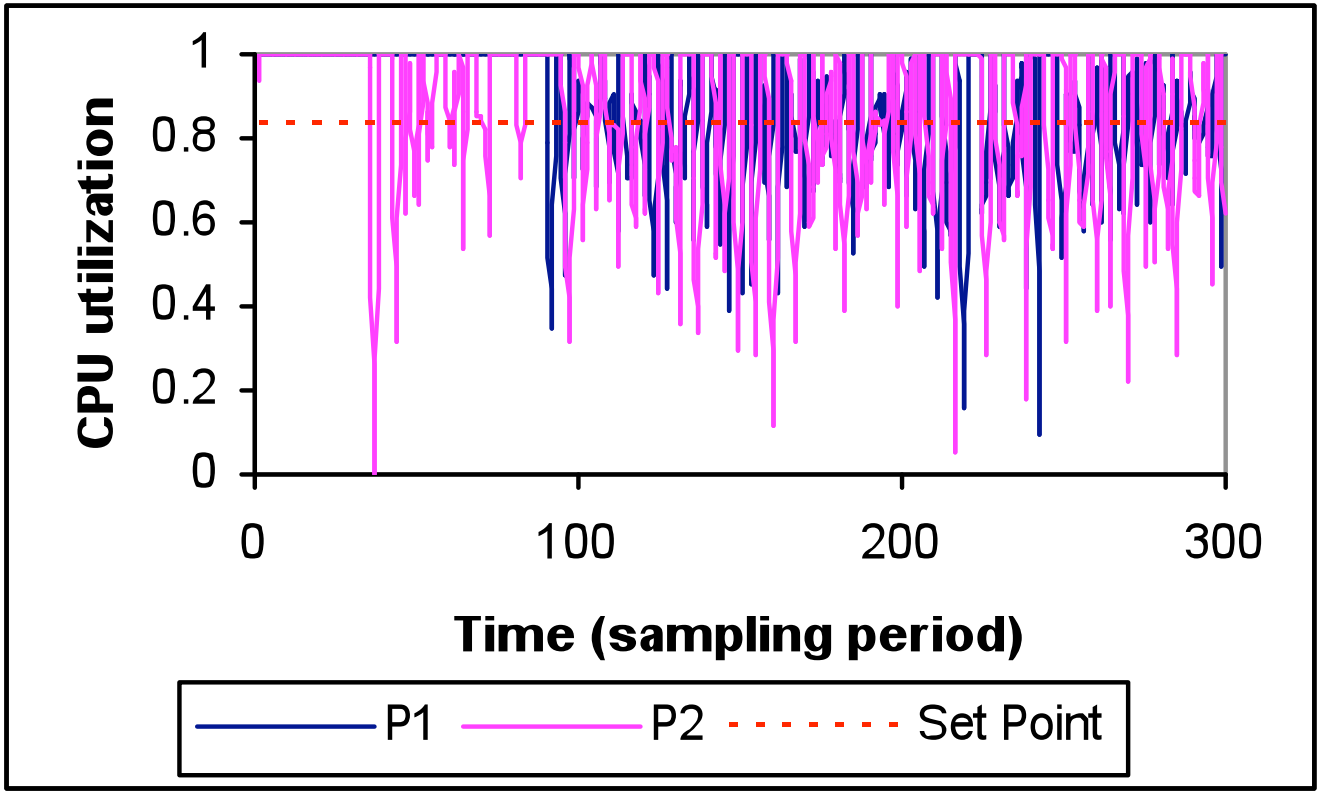
Stable System



execution time factor = 0.5

(actual execution times = ½ of estimates)

Unstable System

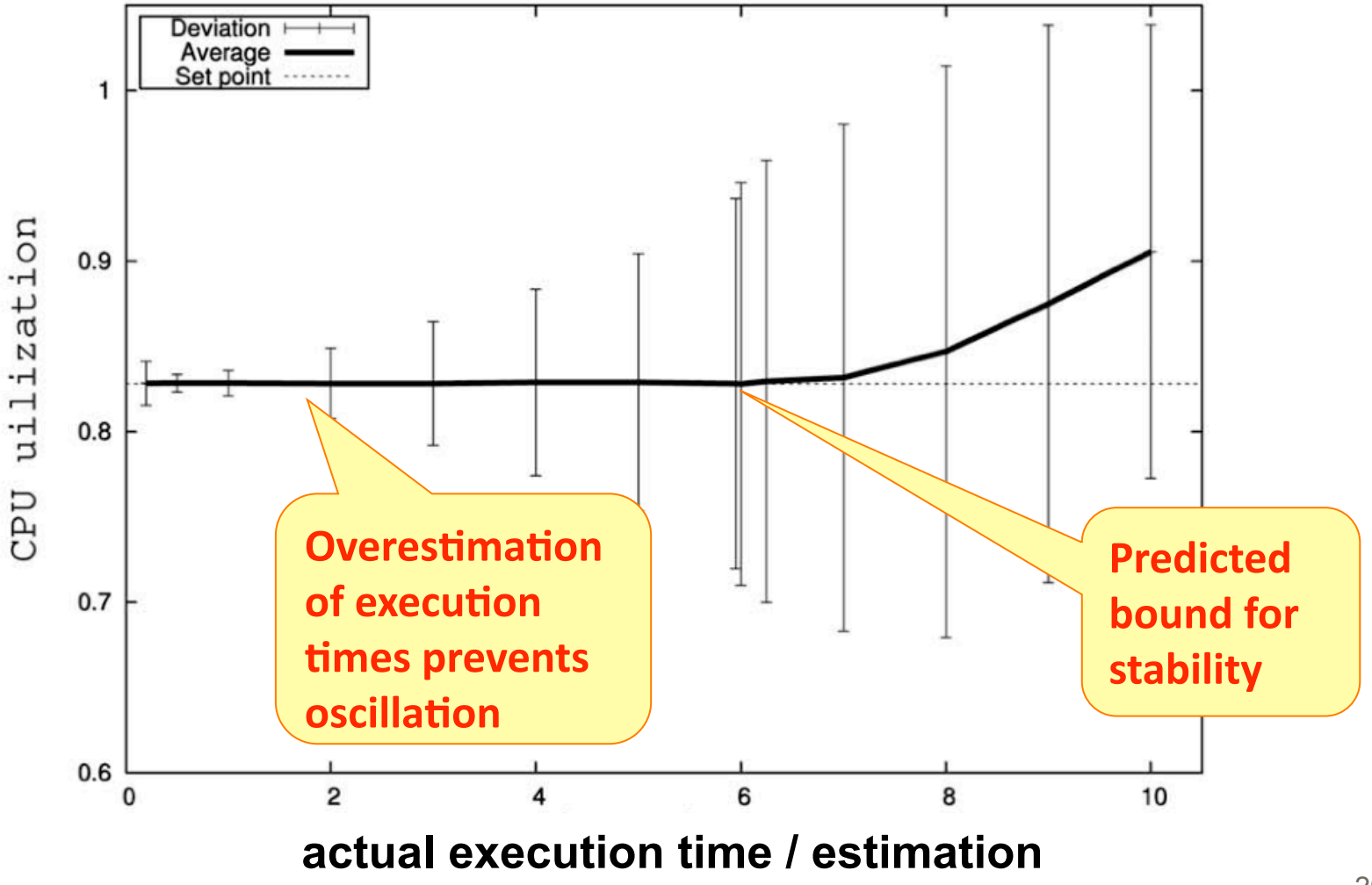


execution time factor = 7

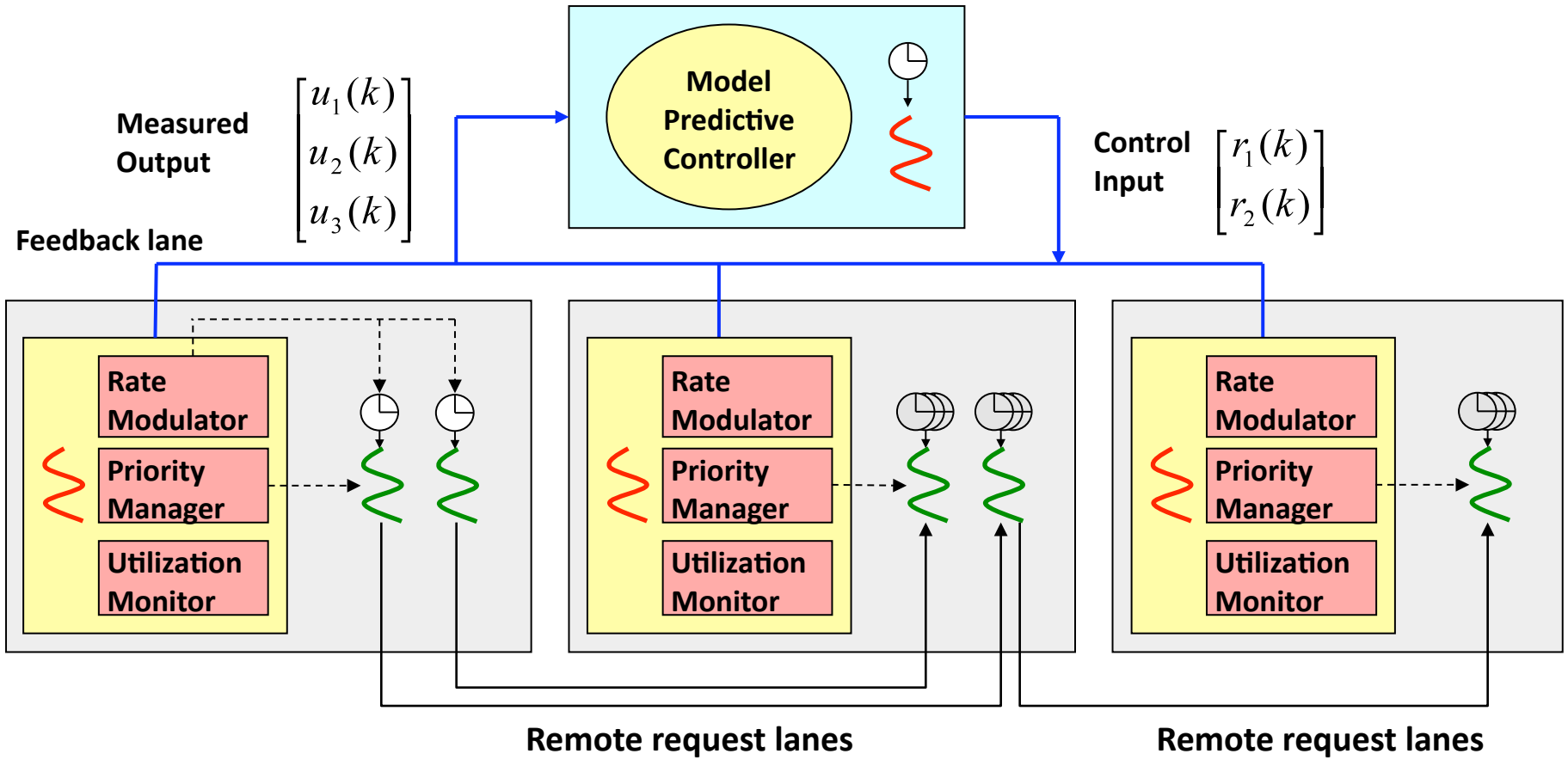
(actual execution times = 7 times estimates)

Stability

- Stability condition → tolerable range of execution times
- Analytical assurance on utilizations despite uncertainty

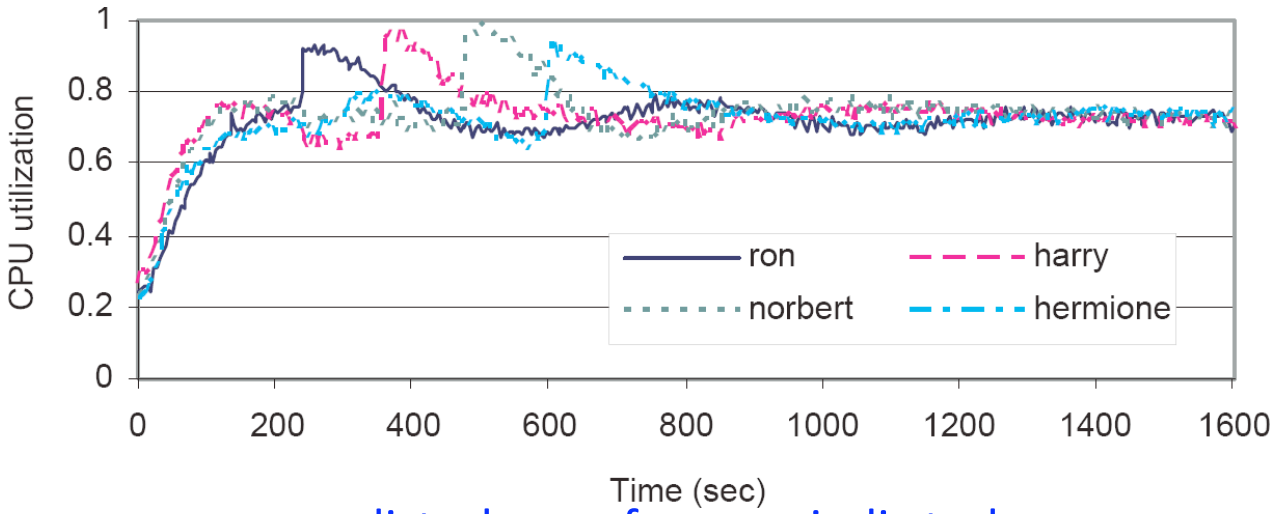


FC-ORB Middleware

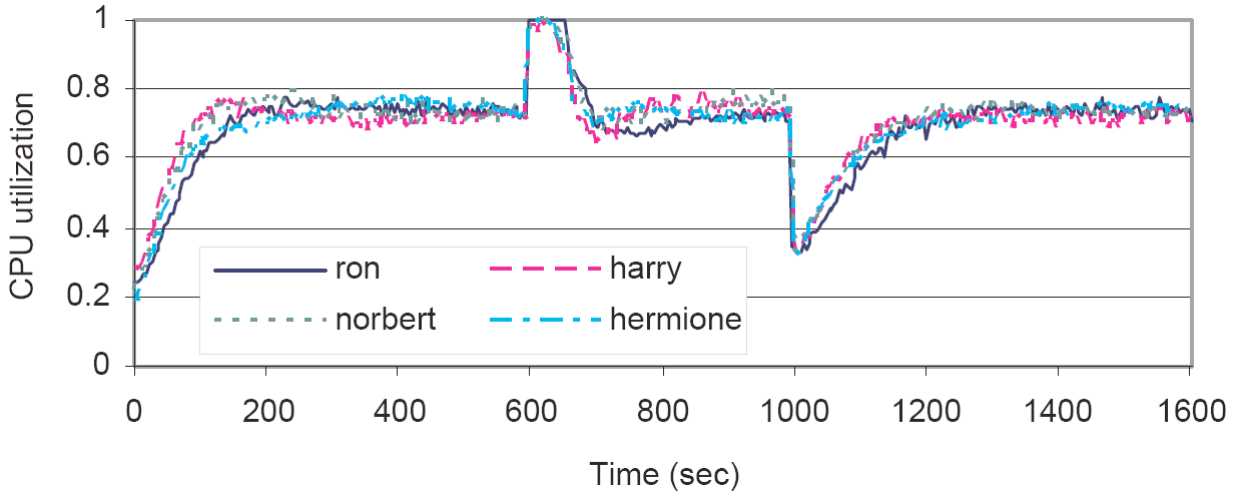


X. Wang, C. Lu and X. Koutsoukos, Enhancing the Robustness of Distributed Real-Time Middleware via End-to-End Utilization Control, IEEE Real-Time Systems Symposium (RTSS'05), December 2005.

Workload Uncertainty

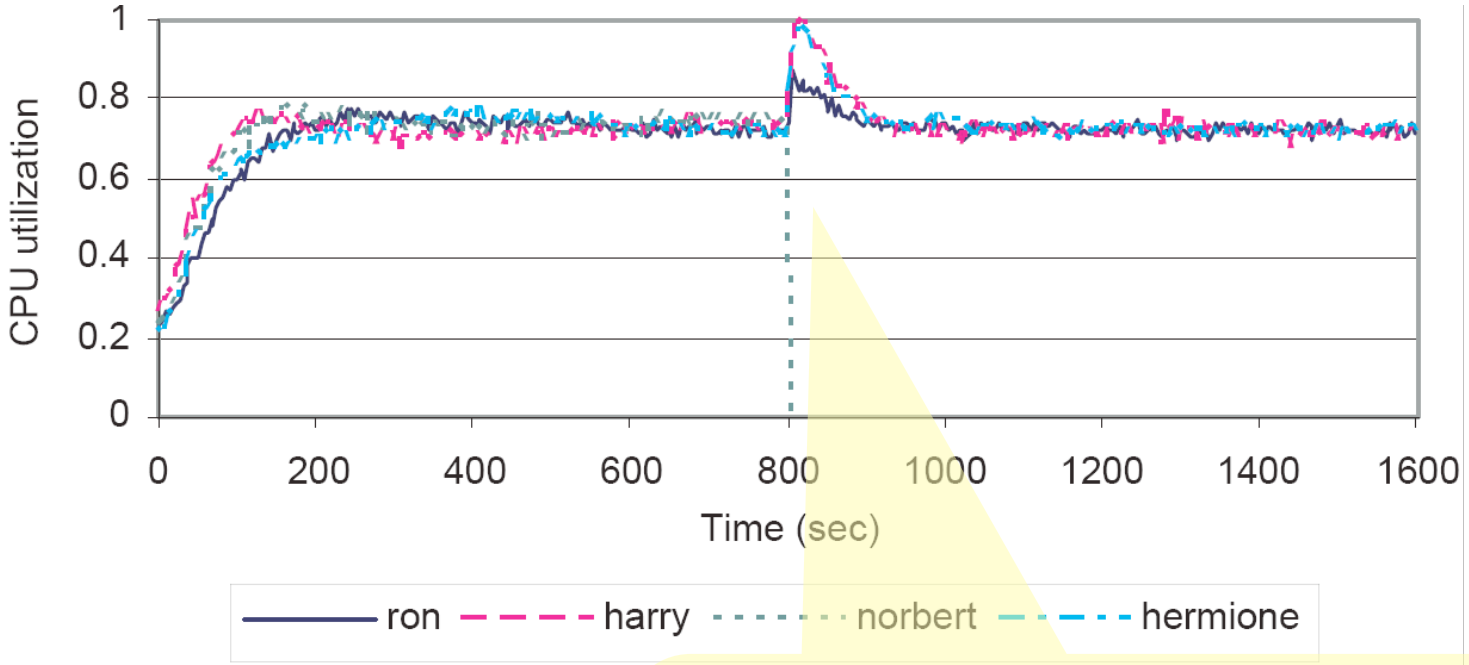


disturbance from periodic tasks



time-varying execution times

Processor Failure



1. Norbert fails.
2. move its tasks to other processors.
3. **reconfigure controller**
4. **control utilization by adjusting task rates**

Summary: Model Predictive Control

- Robust utilization control for distributed systems
- Handle coupling among processors
- Enforce constraints on task rates
- Analyze tolerable range of execution times

References

- **Centralized Control (EUCON):** C. Lu, X. Wang and X. Koutsoukos, Feedback Utilization Control in Distributed Real-Time Systems with End-to-End Tasks, IEEE Transactions on Parallel and Distributed Systems, 16(6): 550-561, June 2005.
- **Middleware (FC-ORB):** X. Wang, C. Lu and X. Koutsoukos, Enhancing the Robustness of Distributed Real-Time Middleware via End-to-End Utilization Control, IEEE Real-Time Systems Symposium (RTSS'05), December 2005.
- **Decentralized Control:** X. Wang, D. Jia, C. Lu and X. Koutsoukos, DEUCON: Decentralized End-to-End Utilization Control for Distributed Real-Time Systems, IEEE Transactions on Parallel and Distributed Systems, 18(7): 996-1009, July 2007.
- **Controllability & Feasibility:** X. Wang, Y. Chen, C. Lu and X. Koutsoukos, On Controllability and Feasibility of Utilization Control in Distributed Real-Time Systems, Euromicro Conference on Real-Time Systems (ECRTS'07), July 2007.
- **Project page: http://www.cse.wustl.edu/~lu/control.html**
- **Model Predictive Control:** J.M. Maciejowski, Predictive Control with Constraints, Prentice Hall, 2002.