CSE 347 Analysis of Algorithms January 30, 2019

Lecture 5 Summary

Guest lecturer: Zihao Deng

1. The Cut Property for Minimum Spanning Tree
   (a) For any Minimum Spanning Tree and any partition of the nodes into two pieces, a minimum cost edge crossing from one piece to the other must be included in the tree.
   (b) Proof is by an *"exchange argument"*—for any tree that does not include such an edge, we can reduce the cost by swapping a minimum cost edge in for some other edge of the tree.
2. Given the Cut Property, the analysis of the correctness of both Prim's Algorithm and Kruskal's Algorithm are straightforward: the final tree will always have to include the chosen edge.
   (a) The proof is easier if we assume the costs are all *distinct*. This is easily ensured by "perturbing" the costs by a negligible amount so that they become distinct, even if they were originally not.