CSE 347 Analysis of Algorithms January 14, 2019

Lecture 1 Summary

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1. Administrivia: see course webpage `http://www.cse.wustl.edu/~bjuba/cse347/s19`
2. Matrix Multiplication via Divide and Conquer
   (a) Matrix multiplication: for $n \times n$ matrices A, B $C = AB$ is the $n \times n$ matrix where $C_{ij} = \sum_{k=1}^{n} A_{ik} \cdot B_{kj}$
   (b) Naïve algorithm is $O(n^3)$
   (c) What if we write $A = \left[\begin{array}{c|c} A^{(1,1)} & A^{(1,2)} \\ \hline A^{(2,1)} & A^{(2,2)} \end{array}\right], B = \left[\begin{array}{c|c} B^{(1,1)} & B^{(1,2)} \\ \hline B^{(2,1)} & B^{(2,2)} \end{array}\right], C = \left[\begin{array}{c|c} C^{(1,1)} & C^{(1,2)} \\ \hline C^{(2,1)} & C^{(2,2)} \end{array}\right]$? Then $C^{(1,1)} = A^{(1,1)}B^{(1,1)} + A^{(1,2)}B^{(2,1)}$, etc.
   (d) Recursive algorithm has 8 subproblems of size $n/2$, gives n^3 time again
3. Strassen's Algorithm
   (a) If we can use seven subproblems instead of eight, we get a running time of $n^{2.81}$
   (b) Strassen's algorithm achieves this with the following subproblems

$$
\begin{aligned}
P_1 &= A^{(1,1)}(B^{(1,2)} - B^{(2,2)}) \\
P_2 &= (A^{(1,1)} + A^{(1,2)})B^{(2,2)} \\
P_3 &= (A^{(2,1)} + A^{(2,2)})B^{(1,1)} \\
P_4 &= A^{(1,2)}(A^{(2,1)} - B^{(1,1)}) \\
P_5 &= (A^{(1,1)} + A^{(2,2)})(B^{(1,1)} + B^{(2,2)}) \\
P_6 &= (A^{(1,2)} - A^{(2,2)})(B^{(2,1)} + B^{(2,2)}) \\
P_7 &= (A^{(1,1)} - A^{(2,1)})(B^{(1,1)} + B^{(1,2)})
\end{aligned}
$$

   combined the following way

$$
\begin{aligned}
C^{(1,1)} &= P_5 + P_4 - P_2 + P_6 \\
C^{(1,2)} &= P_1 + P_2 \\
C^{(2,1)} &= P_3 + P_4 \\
C^{(2,2)} &= P_5 + P_1 - P_3 - P_7
\end{aligned}
$$