CSE 347 Analysis of Algorithms March 27, 2019

Sample Midterm Exam 2

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This is a sample midterm exam. It is intended to be representative of the content, length, and difficulty of the questions you will be asked on your midterm exam.

Instructions. Answer questions as clearly and concisely as you can. Try to fit your solutions in the space provided, continuing on the provided extra sheets of paper if necessary. We will award partial credit based on the work that you provide; if you get stuck, *be sure to explain what you are trying to do.*

This is a *closed book* exam. The use of laptops, phones, smartwatches, etc. is *not allowed.* You are allowed to reference a crib sheet that you prepared yourself, contained entirely on a single 8.5×11" (US letter size) sheet of paper. Any other written material is not permitted.

You may use (without proof) any theorem that has been stated in lecture. Except on the Reading Solutions problem, you may also reference the result of the homework problems you were assigned without proof. You *may not* use the result of other problems from the book without providing a proof.

Name:

Student ID:

Problem Number	Points Possible	Points Received
1	25	
2	25	
3	30	
TOTAL	80	

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1. (a) *(15 points)* Suppose we have a randomized algorithm for an *optimization problem* that only produces an optimal solution with some small probability $\epsilon > 0$. (We assume that we can compute the objective value of the returned solutions in polynomial time using a deterministic algorithm.) Describe how to obtain a polynomial-time algorithm that returns a correct solution with probability $1 - \delta$ for a given $\delta > 0$ (and show that your solution is correct).

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(b) *(10 points)* Suppose we have a randomized algorithm for a (yes/no) *decision problem* that only returns the correct decision with probability $1/2 + \epsilon$ for some small $\epsilon > 0$. In this case, we *do not* have an algorithm to check whether or not the returned answers are correct. Describe how to obtain a polynomial-time algorithm that returns the correct answer with probability $1 - \delta$ for a given $\delta > 0$ (and show that your solution is correct).

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2. *(25 points)* **Reading solutions.** The following is a problem that appeared on your homework. *For this problem only,* you may not simply reference the fact that it appeared on your homework. You must present a solution to this problem.

   You are given a graph $G = (V, E)$ and you want to drive from a location s to another location t without running out of gas while spending as little money as possible. You are given a map of the road system designated by a set of vertices and a set of directed roads (one way) where $(u, v) \in E$ means that there is a road from vertex u to vertex v. $w(u, v)$ is the length of the road from u to v, and it uses up $w(u, v)$ gas units. You also know that some vertices $S \subseteq V$ have gas stations. When you get to a vertex with a gas station, you can either pay $c(u)$ dollars to fill up your tank, or not get gas. (Note that the cost does not depend on the amount of gas you get, only on the gas station location. So you may as well fill up your tank if you are getting any gas.) Assume that you start with a full tank with k units of gas. Give an efficient algorithm to find the cheapest route from s to t such that you are never stranded without gas. (It is ok to run out of gas at the same moment you reach a gas station.) Analyze the running time of your algorithm. Your analysis should be in terms of S, V and E.

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3. *(30 points)* HITTING SET is the following problem: you are given a family of sets $\{S_1, S_2, \ldots, S_n\}$ and a budget k, and you wish to decide if there is a set H of size at most k such that for all i, there is an element in $S_i \cap H$ (H is a *"hitting set"*). Prove that HITTING SET is NP-complete.