CSE 347 Analysis of Algorithms April 24, 2019

Sample Final Exam

Instructor: Brendan Juba *110 minutes*

This is a sample final exam. It is intended to be representative of the content, length, and difficulty of the questions you will be asked on your final exam.

Instructions. Answer questions as clearly and concisely as you can. Try to fit your solutions in the space provided, continuing on the provided extra sheets of paper if necessary. We will award partial credit based on the work that you provide; if you get stuck, *be sure to explain what you are trying to do.*

This is a *closed book* exam. The use of laptops, phones, smartwatches, etc. is *not allowed.* You are allowed to reference a crib sheet that you prepared yourself, contained entirely on a single 8.5×11" (US letter size) sheet of paper. Any other written material is not permitted.

You may use (without proof) any theorem that has been stated in lecture. Except on the Reading Solutions problem, you may also reference the result of the homework problems you were assigned without proof. You *may not* use the result of other problems from the book without providing a proof.

Name:

Student ID:

Problem Number	Points Possible	Points Received
1	25	
2	25	
3	25	
4	35	
TOTAL	110	

1. *(25 points)* **Reading solutions.** The following is a problem that appeared on your homework. *For this problem only,* you may not simply reference the fact that it appeared on your homework. You must present a solution to this problem.

   Let $G = (V, E)$ be an undirected graph with n nodes and m edges. For a subset $X \subseteq V$, we use $G[X]$ to denote the subgraph *induced* on X – that is, the graph whose node set is X and whose edge set consists of all edges of G for which both ends lie in X.

   We are given a natural number $k \leq n$ and are interested in finding a set of k nodes that induces a "dense" subgraph of G; we'll phrase this concretely as follows. Give a polynomial-time algorithm that produces, for a given natural number $k \leq n$, a set $X \subseteq V$ of k nodes with the property that the induced subgraph $G[X]$ has at least $\frac{mk(k-1)}{n(n-1)}$ edges.

   You may give either (a) a deterministic algorithm, or (b) a randomized algorithm that has an expected running time that is polynomial, and that only outputs correct answers.

2. *(25 points)* Your dorm room is in the center of a very long hallway, with $n/2$ evenly-spaced rooms on either side. One of these rooms has free pizza, but you don't know which one. All the doors are open, so you only need to walk past a room to decide if it contains the free pizza. We want to minimize the number of rooms we must walk past before finding the free pizza. Give a 9-competitive algorithm for this problem.

3. *(25 points)* Consider a telecommunications network consisting of a cycle on n nodes, numbered 0 through $n-1$ clockwise around the cycle. Some set C of calls is given; each call is a pair (i, j) originating at node i and destined for node j. The call can be routed either clockwise or counterclockwise around the ring. Our goal is to route the calls so as to minimize the total *load* on the network. The load L_i on link $(i, i+1 \pmod n)$ is the number of calls routed through link $(i, i+1 \pmod n)$, and the total load is

$$max_{1 \leq i \leq n} L_i$$

Give a 2-approximation algorithm for this problem.

4. *(35 points)* Recall the definition of a spanning tree: given an undirected graph $G = (V, E)$, a spanning tree is a connected subgraph $T = (V, E')$, where T is a tree and $E' \subseteq E$.

   Consider the following problem. Given as input a pair (G, k) where G is an undirected, connected graph and k is an integer, decide whether G contains a spanning tree such that all vertices in the tree have degree *at most* k?

   Either prove this problem is NP-Complete or give a polynomial time algorithm to solve it.