On the Design of Revenue-Enhancing Signal Structures in Common- and Interdependent-Value Second Price Auctions

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Abstract. We consider the problem of designing the information environment for revenue maximization in a sealed-bid second price auction with two bidders. Much of the prior literature has focused on signal design in settings where bidders are symmetrically informed, or on the design of optimal mechanisms under fixed information structures. We study common- and interdependent-value settings where the mechanism is fixed (a second-price auction), but the auctioneer controls the signal structure for sellers. We show that in a standard common-value auction setting, there is no benefit to the auctioneer in terms of expected revenue from sharing information with the bidders, although there are effects on the distribution of revenues. In an interdependent-value model with mixed private- and common-value components, however, we show that asymmetric, information-revealing signals can increase revenue.

1 Introduction

In most of the literature on mechanism design, the model assumes that agents' information is given, and searches for rules of the game that yield desired outcomes. However, there has recently been considerable interest in the parallel problem of designing the *information environment* that agents will encounter [17,9]. This paradigm is clearly applicable in many scenarios of interest to AI researchers, including online advertising, internet marketplaces, and so on. One particular domain where this is interesting is in auctions with signaling, which have been studied extensively in both economics and computer science. Assume a fixed mechanism; can the seller expect to make more revenue if the bidders are more or less informed than the "baseline"?

Auctions with signaling have been studied in several different contexts. Much of the literature assumes that agents are symmetric with respect to the information they receive about the value of the item, in the sense that the bidders' signals are drawn from the same distribution. For example, the seminal "Linkage Principal" of Milgrom and Weber [22] states that fully and publicly announcing all information available to the seller is the expected-revenue-maximizing policy in common value auctions. Somewhat less is known about auctions with asymmetrically informed bidders, and most of that literature has focused on understanding how information asymmetries affect revenue rather than on the design of the optimal signal structure. There has also been a line of work on so-called "deliberative auctions" [21,5], where agents have the opportunity to acquire information about valuations before entering a bidding process. Most of this literature focuses on strategic choices by the bidders and how this affects equilibrium outcomes of the auction.

Here we analyze signal design in auctions as a persuasion game, following Kamenica and Gentzkow [17] who consider the problem of designing the optimal information environment for the case between one self-interested agent ("sender") and one decision-maker ("receiver"), where both of them are rational Bayesians. The sender can design the *information structure* or *signal structure* to release information about the state of the world to receiver before the receiver makes her choice. In the auction setting, the signal structure induces a game between the bidders, and the equilibrium outcome of the game affects the seller's revenue.

Our contribution We consider a sealed-bid second price auction with two bidders. As usual, the winner is the bidder who submitted the highest bid (with ties broken equiprobably in either direction), but pays to the seller the second highest bid. The bidders and seller share the same common prior on the underlying state of the item. Before the bidding stage, the seller can provide a (noisy) signal to each bidder based on the state of the world. She commits to a signaling strategy in advance, and the resultant structure becomes common knowledge. We explore the following two auction games: (1) a basic common-value auction model, where the value of the item is determined either by a single attribute or by two independent attributes when each bidder can receive information from eactly one of the attributes; (2) an interdependent-value auction, where the valuation for each bidder is decided by a common value attribute and a private attribute. We show that in the common-value auction settings, there is no benefit to the auctioneer in terms of expected revenue from sharing information with the bidders, although there are effects on the distribution of revenues. In an interdependent-value model with mixed private- and common-value components, however, we show that asymmetric, information-revealing signals can increase revenue.

1.1 Related work

Our work is related to several literatures. Broadly, this paper fits into a growing line of literature in AI on how the information environment available to agents influences market outcomes. Hajaj and Sarne [15] examine how e-commerce platforms can gain from information withholding policies. Chhabra *et al* [8] study the welfare effects of competition between information providers with different levels of information quality. Das and Li [10] model the effects of common and private signals about quality in matching with interviews. Rabinovich et al [25] present an efficient model for security asset assignment which combines both Stackelberg security games and the Bayesian Persuasion model.

The literature on auctions with signaling, as mentioned above, typically analyzes symmetric information structures, where there are few positive results in terms of revenue enhancements. In addition to the literature from economics cited above, recent work in algorithmic economics that assumes symetric information disclosure includes that of Emek et al [12] as well as Bro Miltersen and Sheffet [6], both of which study second-price auctions of multiple indivisible goods and consider hiding information by clustering. Guo and Deligkas [14] single-item second-price auctions where the item is characterized by a set of attributes and the auctioneer decides whether to hide a subset of attributes.

When we move to asymmetric information, most early work considers the case in which one bidder is perfectly informed about the value of the item, while the other bidders are entirely uninformed [30,23]. Milgrom and Weber [23] show that reducing information asymmetries can increase the seller's expected revenue in a two-bidder first-price common value auction where one bidder is perfectly information and the other bidder is entirely uninformed. Goeree and Offerman [13] also consider public information disclosure in common value auctions, in which the common value is an average of i.i.d. private values (signals) of all bidders. They also conclude that seller's public information disclosure can raises efficiency and seller's revenues. Hausch [16], however, through a simple example in a first price common value auction, shows that reducing information asymmetry may decrease the seller's expected revenue when the better-informed bidder is neither strictly better-informed nor perfectly informed.

Syrgkanis *et al* [27] consider common value hybrid auctions where the payment is a weighted average of the highest and second-highest bids. They show that public revelation of an additional signal to both bidders may decrease the auctioneer's revenue, different from [22]. Parreiras [24] consider continuous signal spaces and also show that second price auction revenue-dominates first price auction. In both of these papers, the seller does not control the information structure for both bidders.

There are also several recent papers considering this question from the optimal mechanism design perspective [26,3,11], rather than assuming a fixed structure for the mechanism and analyzing the question of optimal signaling given the mechanism. Very recent work of Alkoby et al [2] analyzes signaling by a third party information provider under a fixed mechanism.

Also related is the literature on deliberative auctions. Deliberation covers any actions that update an agent's belief. In the study of deliberative auctions, research has thus far focused on either the perspective of bidders (receivers) or on optimal mechanism design. Larson and Sandholm [19,20] provide a very general model for costly information gathering in auctions. They show that under costly deliberation, bidders perform strategic deliberation in equilibrium in most standard auction settings (Vickrey, English, Dutch, first price and VCG). Thompson and Leyton-Brown [28] investigate deliberation strategies for second price auctions where agents have independent private values (IPV) and the impact of agents' strategies on seller's revenue. They perform equilibrium analysis for (1) deliberation with costs, (2) free, but time-limited deliberation. They further show that, in the IPV deliberative-agent setting, the only dominant-strategy mechanism is a sequential posted price auction, in which bidders are sequentially given a posted-price, take-it-or-leave-it offer until the good is sold [29]. Celis *et al* [7] provide an efficient mechanism in IPV deliberative-agent setting to obtain revenue within a small constant factor of the maximum possible revenue. Brinkman *et al* [5] show that the dependence structures among agents' signals of the value of the item they are bidding on can produce qualitatively different equilibrium outcomes of the auction. This literature also typically does not focus on the optimal design of the signal structure from the perspective of the seller.

2 Common Value Auctions

We begin by considering a single-item auction with two risk-neutral bidders (agents) $i \in \{1, 2\}$ and a seller. Both bidders value the object identically: the item has a common value of $v \in R^+$ to the two bidders. The realization of v is not observed by either the seller or the bidders. v depends on an underlying state of the world $w \in \Omega$. Without loss of generality, we assume that the item's value is 0 when w's quality is *Bad (B)* and 1 when w's quality is *Good (G)*, and the common prior is represented by $P(G) = x, x \in [0, 1]$. A signal consists of a finite realization space S and a family of distributions $\{P[s|w]\}_{w \in \Omega}$. Kamenica and Gentzkow [17] show that there exists an optimal signal structure with $|S| \leq |\Omega|$.[1] Thus, we only need to consider a binary signal space. Before bidding, each bidder receives a conditionally independent low (L), or high (H) signal from seller without cost, $s_i \in \{H, L\}$.

$$P[s_1 = H|G] = p_1 \; P[s_1 = L|B] = q_1$$
$$P[s_2 = H|G] = p_2 \; P[s_2 = L|B] = q_2$$

where s_i is agent i's signal and all signals have accuracy of $p_i, q_i \in [1/2, 1]$. Thus, a high (low) signal suggests a good (bad) value of the item.

Following prior literature, we make some assumptions.

Assumption 1 *Seller cannot distort or conceal information once the signal realization is known. [17].*

Assumption 2 *Bidders play only weakly undominated strategies. [5]*

The first assumption allows us to abstract from the incentive compatibility issues, while the second helps rule out implausible or uninteresting equilibria.

In the game, the seller decides the signal structure S with the goal of maximizing her expected revenue R and the bidders submit their bids based on their private signals s_i. The seller runs a two-player second-price sealed-bid (SPSB) auction. Define $\text{bid}_{s_{-i}}(s_i)$ as the bid of bidder i given she receives signal s_i and the other bidder receives signal s_{-i}. The seller can either reveal the realization of the signal privately to the corresponding bidder, or reveal it publicly. Here we show the analysis of private revelation, as public revelation follows similarly.

Proposition 1. *If the seller reveals the realization of the signal privately to the corresponding bidder, a unique symmetric equilibrium exists. Each agent bids her expected value conditioned on her opponent's signal being equal to her own,*

$$\begin{aligned} bid_L(L) &= \mathbb{E}[v|s_1 = L, s_2 = L] \\ &= P(G|s_1 = L, s_2 = L) \\ &= \frac{(1-p_1)(1-p_2)x}{(1-p_1)(1-p_2)x + q_1 q_2 (1-x)}, \end{aligned}$$

[1] Specifically, they show that $|S|$ need not exceed $\min\{|A|, |\Omega|\}$ where A is the action space (Proposition 4 in the online appendix).

$$\begin{aligned} bid_H(H) &= \mathbb{E}[v|s_1 = H, s_2 = H] \\ &= P(G|s_1 = H, s_2 = H) \\ &= \frac{p_1 p_2 x}{p_1 p_2 x + (1-q_1)(1-q_2)(1-x)}. \end{aligned}$$

The proof of this proposition is similar to prior work of Hausch [16] and of Brinkman, Wellman, and Page [5], and can be found in the appendix.

Equilibrium selection It is well known that the second-price common-value auction generally has many equilibria [18,16,1, and so on]. Assumption 2 helps us to rule out all dominated bids. In this game, suppose that Bidder 1 obeys the strategy in Proposition 1. Bidder 2, conditional on receiving signal L bids $b \in (\text{bid}_L(L), \mathbb{E}[v|s_1 = H, s_2 = L]]$ and, conditional on receiving signal H, bids $\text{bid}_H(H)$. These strategies are still Nash equilibria. Thus, Nash equilibrium provides no prediction about revenue beyond an upper bound on the full surplus. For this paper's purpose, therefore, we only focus on symmetric equilibrium bidding strategies.

In a common value auction, the seller's expected revenue R is the expected value $\mathbb{E}[v]$ of the item, minus the sum of the two bidders' utilities. When each bidder observes a private signal only, we can treat each bidder independently and minimize the utility of each bidder.

Theorem 1. *If each bidder observes her own private signal, the optimal signal structure for the seller in terms of revenue is* $p_1 = p_2 = 1, q_1, q_2 \in [1/2, 1]$, *or* $p_1 = p_2 = q_1 = q_2 = 1/2$, *where* $\max R = \mathbb{E}[v]$.

Proof. For revenue maximization, we can treat the two-bidder second-price sealed-bid auction as a three-player, constant-sum game. The revenue

$$R = \mathbb{E}[v] - \mathbb{E}[u_1] - \mathbb{E}[u_2]. \tag{1}$$

$\mathbb{E}[u_i]$ is Bidder i's expected utility and R is maximized when $\mathbb{E}[u_1] = \mathbb{E}[u_2] = 0$, where

$$\begin{aligned} \mathbb{E}[u_1] &= p(s_1 = H, s_2 = L)(\mathbb{E}[v|s_1 = H, s_2 = L] - \text{bid}_L(L)) \\ &= p_1(1-p_2)x - p(s_1 = H, s_2 = L)\text{bid}_L(L), \\ \mathbb{E}[u_2] &= p(s_1 = L, s_2 = H)(\mathbb{E}[v|s_1 = L, s_2 = H] - \text{bid}_L(L)) \\ &= (1-p_1)p_2 x - p(s_1 = L, s_2 = H)\text{bid}_L(L), \end{aligned}$$

which gives us

$$p_1 = p_2 = 1, q_1, q_2 \in [\frac{1}{2}, 1], \forall x \in [0, 1],$$

or

$$p_1 = p_2 = q_1 = q_2 = \frac{1}{2}, \forall x \in [0, 1],$$

or

$$p_1, p_2, q_1, q_2 \in [\frac{1}{2}, 1], \text{when } x = 1.$$

When $p_1 = p_2 = q_1 = q_2 = 1$, the seller always reveals complete information, thus the expected revenue R is also $\mathbb{E}[v]$. □

We can see that there is a wide range of signal structures that achieve the maximum revenue in equilibrium, and none of these is better than a policy of revealing no information at all. Another natural question to ask concerns the distribution of revenues to the seller under different signal structures. It is relatively easy to compute the variance of the revenue

$$var(R) = (\text{bid}_L(L) - \text{bid}_H(H))^2(1 - P(HH))P(HH) \tag{2}$$

Clearly $var(R)$ is minimized at $q_1 = q_2 = 0.5$. Figure 1 shows some illustrative examples of the standard deviation of revenue.

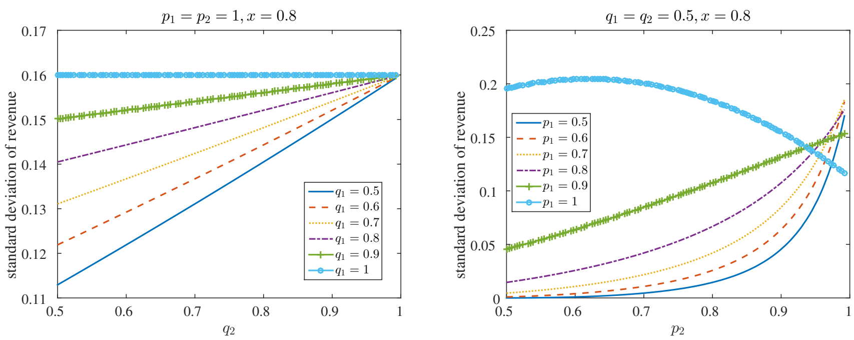

Fig. 1. Standard deviations of revenue for different revenue-maximizing signal structures in the simple common-value model. While each of these signal structures achieves the same revenue, the risk profiles are substantially different.

2.1 Adding an Intermediate Value

Brinkman *et al* [5] study a common-value auction setting with intermediate values, which serves as a model for studying signal acquisition by bidders. They motivate this setting with an example of the auction of extraction rights for some resources (say oil and gas) on a specified plot of land. The value to energy companies of these rights depends on the unknown amounts of extractable resources. The question of optimal signaling is motivated in this example by the fact that the government can reveal information about one or both of the specific resources to each energy company. Now the item can take on three possible values, $\{0, g, 1\}$ with $g \in [0, 1]$. The underlying state w which decides the value of the item now has two attributes, $w = (w_1, w_2)$. Each attribute is associated with signals potentially observed by the respective agents. Each bidder can request one signal with no cost. Here we study a variant where the seller can decide which attribute to signal to each bidder and what the corresponding signal structure should be.

Each attribute is still either *Good (G)* or *Bad (B)*, where $P(w_j = G) = x \in [0, 1], j \in \{1, 2\}$. The realization of each signal is also *High (H)* or *Low (L)*. The signal

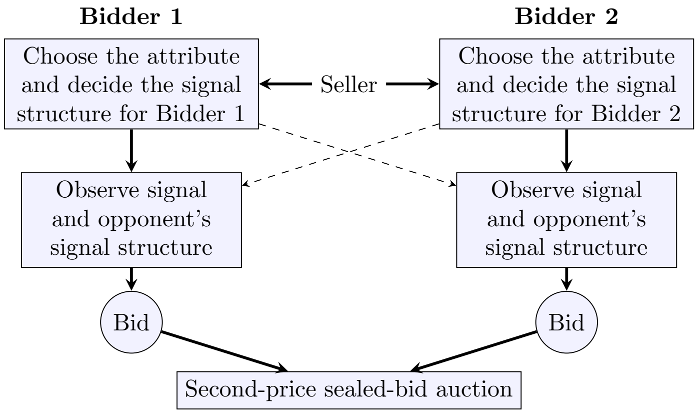

Fig. 2. The intermediate-value model. Dashed lines mean that the bidder knows the structure of the signal that the other bidder receives, but not the specific realization.

structure can be represented as ($s_i^j \in \{H, L\}$):

$$P[s_1^j = H|w_j = G] = p_1 \; P[s_1^j = L|w_j = B] = q_1$$
$$P[s_2^j = H|w_j = G] = p_2 \; P[s_2^j = L|w_j = B] = q_2$$

where $j \in \{1, 2\}$ and s_i^j is Bidder i's signal from attribute j. All signals have accuracy of $p_i, q_i \in [1/2, 1]$.

The value of the good is 0 if neither attribute is G, 1 if both are G, and $g \in [0, 1]$ if only one is G.

$$v = \begin{cases} 0, \text{if } \sum_j \mathbb{I}\{w_j = G\} = 0 \\ g, \text{if } \sum_j \mathbb{I}\{w_j = G\} = 1 \\ 1, \text{if } \sum_j \mathbb{I}\{w_j = G\} = 2 \end{cases}$$

Figure 2 shows the decision flow in this game. The seller's goal is to maximize her expected revenue R. The signal structure and the seller's choice of which attribute to signal to each bidder are both common knowledge.

First, we observe that it must again be the case that the seller's revenue is maximized when revealing no information even in this intermediate value setting, since it can still be modeled as a three-player, constant-sum game, and Equation (1) holds. What can we say about signal structures that achieve this revenue? Again, we analyze private revelation.

Theorem 2. *In the intermediate value model, (1) if the seller sends signals of different attributes to the two buyers, there is only one signal structure,* $\forall g, x \in [0, 1]$, $p_1 = p_2 =$

$q_1 = q_2 = 1/2$ (equivalent to sending no information) that achieves the maximum possible revenue; (2) if the seller sends signals of the same attribute to both buyers, for $\forall g, x \in [0, 1]$, there are a number of signal structures that achieve the maximum possible revenue: $p_1 = p_2 = 1, q_1, q_2 \in [1/2, 1]$ or $p_1 = p_2 = q_1 = q_2 = 1/2$.

Proof. The seller's revenue still follows Equation (1). To maximize $R, \mathbb{E}[u_1] = \mathbb{E}[u_2] = 0$.

- *Sending signals of the same attribute:*
  The unique symmetric equilibrium bidding strategy is that each bidder bids her expected value conditioned on her opponent's signal being equal to her own,

$$\text{bid}_L(L) = \mathbb{E}(v|s_i^j = L, s_{-i}^j = L),$$
$$\text{bid}_H(H) = \mathbb{E}(v|s_i^j = H, s_{-i}^j = H).$$

  We denote $P(s_i^j = H, s_{-i}^j = L)$ by $P(HL)$,

$$\mathbb{E}[u_i] = P(HL)(\mathbb{E}(v|s_i^j = H, s_{-i}^j = L) - \text{bid}_L(L)).$$

  Thus, to maximize R

$$\mathbb{E}(v|s_i^j = H, s_{-i}^j = L) = \text{bid}_L(L). \tag{3}$$

  The solution of Equation (3) is

$$p_1 = p_2 = q_1 = q_2 = \frac{1}{2}, \forall g, x \in [0, 1],$$

  or

$$p_1 = p_2 = 1, q_1, q_2 \in [\frac{1}{2}, 1], \forall g, x \in [0, 1],$$

  or

$$p_1, p_2, q_1, q_2 \in [\frac{1}{2}, 1], \text{when } x = 1, \forall g \in [0, 1].$$

  When $p_1 = p_2 = q_1 = q_2 = 1$, the seller reveals perfect information, thus the expected revenue R is also $\mathbb{E}[v]$.
- *Sending signals of different attributes:*
  As the signal accuracy between different attribute is identical, the equilibrium bidding strategy is same as above, that is to bid the expected valuation conditioned on the opponent observing the same signal value. Denote $\text{bid}_{-s_{-i}}(s_i)$ as the bid of Bidder i given she receives s_i and the other bidder observes the signal of the other attribute and receives signal s_{-i} ,

$$\text{bid}_{-L}(L) = \mathbb{E}(v|s_i^j = L, s_{-i}^{-j} = L),$$

$$\text{bid}_{-H}(H) = \mathbb{E}(v|s_i^j = H, s_{-i}^{-j} = H).$$

  We simplify $P(s_i^j = H, s_{-i}^{-j} = L)$ by $P(H, L)$,

$$\mathbb{E} = [u_i] = P(H, L)(\mathbb{E}(v|s_i^j = H, s_{-i}^{-j} = L) - \text{bid}_{-L}(L)).$$

Thus, to maximize R,

$$\mathbb{E}(v|s_i^j = H, s_{-i}^{-j} = L) = \text{bid}_{-L}(L). \tag{4}$$

Solving Equation (4) we get,

$$p_1 = p_2 = q_1 = q_2 = \frac{1}{2}, \forall g, x \in [0, 1].$$

□

It is again easy to show that $var(R)$ is minimized at $q_1 = q_2 = 0.5$.

Discussion Brinkman *et al* [5] analyze this problem from the perspective of the bidders. In their model, the signal structure is fixed and restricted to the symmetric information case ($p_1 = p_2 = q_1 = q_2$). They show that when the two attributes are sufficiently complementary, that is $g \to 0$, and the signals are noisy, the agents choose to observe the same attribute. When the signal accuracy is high, or the two signals are substitutable $g \to 1$, the agents choose to observe different attributes. Our result above demonstrates that, from the seller's perspective, sending no information can always maximize seller's expected revenue. The seller can also achieve the maximum possible revenue by sending information on the same attribute to both bidders. The corresponding signal structure shows that the bidders always know the item is bad if they see a *low* signal, but they have uncertainty when they see a *high* signal.

3 An Interdependent Value Auction

We now move to a setting with an unambiguously positive result for the seller. We consider a classic situation in corporate mergers. A firm (target) can generate synergies if acquired by another firm (bidder) [4]. The source of this synergy may include management, economies of scale, technological matches, tax savings, etc. A sketch of the game is shown in Figure 3. The target's quality can be either good or bad, which is unknown to the market and the bidders at the time of bidding. The bidders' types can be high or low tech, privately known to each bidder. The ability of a bidder to generate synergies can be either high or low, which is unknown to the market and to the bidders, but may be discovered by the target (since the target is willing to invest in discovering this prior to making it known that it is open to acquisition). If the type of a bidder is high tech, as long as the ability of the bidder to generate synergies is high, it can get high value ($\alpha > 1$) no matter the target's quality. However, if the type of a bidder is low tech, only when both the ability of the bidder to generate synergies is high and the quality of the target is good, can it get medium value (1).

3.1 Model

We first extend the common-value model of Brinkman *et al* to this situation. The item's value still depends on an underlying state w, which now has three attributes $w = (w_0, w_1, w_2)$. The common attribute w_0 can affect the valuation of both bidders (quality

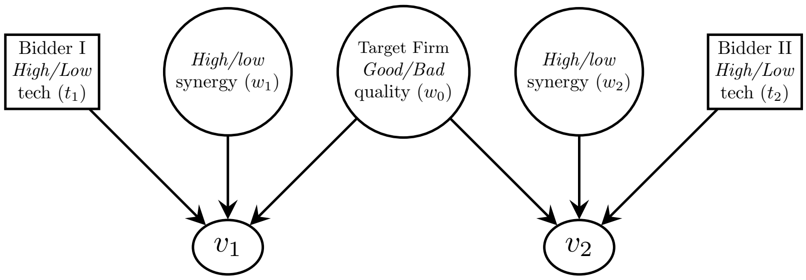

Fig. 3. A sketch of the interdependent value setting.

of the target firm), and the private attributes w_1 and w_2 only affect each bidder's own valuation respectively (idiosyncratic synergies). Each attribute takes quality *Good (G)* or *Bad (B)* as above. For simplicity, we assume $P(w_j = G) = x \in [0, 1], j \in \{0, 1, 2\}$ (this assumption can be easily removed and all results hold). The seller sends a signal of the quality of either common or private attribute w_j to each bidder. The realization of each signal is also *High (H)* or *Low (L)*. The signal structure is ($s_i^j \in \{H, L\}$)

$$P[s_1^j = H|w_j = G] = p_1,\, P[s_1^j = L|w_j = B] = q_1,$$
$$P[s_2^j = H|w_j = G] = p_2,\, P[s_2^j = L|w_j = B] = q_2.$$

All signals have accuracy of $p_i, q_i \in [1/2, 1]$. Once the signal structure is decided, it becomes common knowledge. The seller can choose to either reveal realizations publicly or privately.

The bidders can be of two types, $t_i \in \{t_l, t_h\}$. The bidders will be of either type with probability $P(t_i = t_l) = P(t_i = t_h) = \frac{1}{2}$. If the bidder is type t_h (high tech firm), then her valuation is only dependent on her private attribute, that is $w_i = G$ with value $\alpha > 1$ (pure strategy Nash equilibrium is not guaranteed if $\alpha = 1$) and $w_i = B$ with value 0. If the bidder is type t_l (low tech firm), her valuation is dependent on both common and private attributes: the bidder's value is 0 if both the common and her private attribute are B, and 1 if both are G. Formally,

$$i \in \{1, 2\}$$
$$v_i(w_0, w_i, t_i = t_l) = \begin{cases} 1, \text{ if } w_0 = G, w_i = G, \\ 0, \text{ else}, \end{cases}$$
$$v_i(w_0, w_i, t_i = t_h) = \begin{cases} \alpha, \text{ if } w_i = G, \\ 0, \text{ else}, \end{cases}$$

where $P(t_i = t_l) = \frac{1}{2}$.

3.2 Analysis

Before the game, the seller needs to decide which attribute she wants to signal to each bidder and whether the realization of the signal is public or private. The seller still provides one signal to each bidder, but the realization of that signal can be public. The complete results characterizing the best possible revenue impact and the corresponding signal structure based on seller's strategy is shown in Figure 4. The main results to note are that there are two signal structures that are revenue enhancing. For brevity, we defer the relatively simple proofs of the negative results in cases one through eight to a longer version of this paper, and focus on the two positive outcomes.

When we allow one bidder (w.lo.g. Bidder 2) to observe a signal of her private attribute while the other bidder receives a private signal of the common attribute (case 9), there exists a revenue-enhancing signal structure. In equilibrium, a bidder of type t_h always bids her expected value given the signal realization of private attribute if she receives one. If Bidder 1 is type t_l she bids her expected value given the signal realization she observes. If Bidder 2 is type t_l, if she observes a *low* signal, her bid falls in the range $[\mathbb{E}[v|s_1^0 = L, s_2^2 = L], \mathbb{E}[v|s_1^0 = H, s_2^2 = L]]$ under Assumption 2 and also needs to be smaller than Bidder 1's expected value given Bidder 1 observes a *low* signal $\mathbb{E}[v|s_1^0 = L]$; if she observes a *high* signal, from Assumption 2 her bid falls in the range $[\mathbb{E}[v|s_1^0 = L, s_2^2 = H], \mathbb{E}[v|s_1^0 = H, s_2^2 = H]]$, and also needs to be greater than bidder 1's expected value given Bidder 1 observes a *high* signal $\mathbb{E}[v|s_1^0 = H]$.

Now, suppose the seller chooses signal structure $p_1 \in [0.5, 1], p_2 = 1, q_1 = 1, q_2 = 0.5$. If Bidder 1 observes a high signal, she knows with certainty that the common attribute is good, and is uncertain otherwise. Bidder 2 knows that her private attribute is bad if she observes a low signal, and is uncertain otherwise. Combined with the observation about bid ranges above, it now becomes a simple matter of algebra to show that the expected revenue is greater than that which is achieved when the seller reveals no information or full information, yielding the following theorem:

Theorem 3. *Privately revealing the realization of the common attribute signal to one bidder and privately revealing the realization of the private attribute signal to the other bidder, the seller's expected revenue at* $p_1 \in [0.5, 1], p_2 = 1, q_1 = 1, q_2 = 0.5$ *is always better than that she can achieve when revealing no information or full information.*

An interesting observation about this signal structure is that, while the signal structure conveys more information to Bidder 1, her utility is actually lower compared with when there is no information. Bidder 2's utility improves.

Finally, we see what happens if the seller signals the common attribute to each bidder privately (case 10 in Figure 4). In this situation, the equilibrium bidding strategy for t_h type bidder is to bid her expected value regardless of the signal she receives and for t_l type bidder is to bid her expected value conditioned on the other bidder observing same signal. It is easy to show that the signal structure $p_1 = p_2 = 1, q_1 = 1, q_2 = 0.5$, results in higher expected revenue than when the seller conveys no information or full information.

Theorem 4. *When revealing the signal realization of the common attribute privately to each bidder, the seller's revenue is higher at signal structures* $p_1 = p_2 = 1, q_1 =$

$1, q_2 = 0.5$, *or* $p_1 = p_2 = 1, q_1 = 0.5, q_2 = 1$, *than when revealing no information or full information.*

Consider signal structure $p_1 = p_2 = 1, q_1 = 1, q_2 = 0.5$ (the other one is symmetric). Bidder 1 always has perfect information. If Bidder 2 receives a low signal, she is certain w_0 is bad; however, she is uncertain when she gets a high signal. Surprisingly, although Bidder 1 has perfect information, her expected utility is actually lower than that of Bidder 2. It is easy to see that if both bidders are t_l types or t_h types, then the expected utility of each bidder is zero. The interesting case is when one bidder is a t_h type, and the other one is a t_l type. In this situation, the bidder with imperfect information is more likely to receive a high signal than the bidder with perfect information; therefore, in expectation, the perfect information bidder will pay more (since it is a second price auction), hurting her utility.

Case	c_1	pr_1	c_2	pr_2	Revenue impact	Maximizing structure	Remarks
1	no	no	no	no	–	–	
2	no	yes	no	no	$\downarrow$	no information	unique eq
3	no	yes	no	yes	$\downarrow$	no information	unique eq
4	publicly	no	no	no	–	any	unique eq
5	publicly	no	publicly	no	–	any	unique eq
6	publicly	no	no	yes	$\downarrow$	private signal no information	unique eq
7	publicly	no	privately	no	$\downarrow$	lower bound maximized at no information	multiple eqs
8	privately	no	no	no	$\downarrow$	lower bound maximized at no information	multiple eqs
9	privately	no	no	yes	$\uparrow$	lower bound better than no information	multiple eqs
10	privately	no	privately	no	$\uparrow$	$p_1 = 1, p_2 = 1,$ $q_1 = 1, q_2 = 0.5$	unique symmetric eq

Fig. 4. Best possible revenue impacts and corresponding signal structures in the interdependent value setting. c_i indicates signaling the common attribute to Bidder i and pr_i indicates signaling the private attribute to Bidder i. For the common attribute, "publicly" means the realization of the signal can be observed by all bidders and "privately" means the realization of the signal can only be observed by the corresponding bidder. Since private values are independent, whether that signal is revealed publicly or privately makes no difference. Note that the order of the two bidders is arbitrary, but the existence of the asymmetry is not.

4 Conclusion

We consider information design, or persuasion, in simple auction models. We demonstrate that, while different signal structures may not help improve revenue in second-price sealed bid common value auctions, there are natural auction models, like the interdependent value model for corporate takeovers we present, in which the optimal design of signal structures can be revenue enhancing.

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6 Appendix

6.1 Proof of Proposition 1

Proposition 1 *If the seller reveals the realization of the signal privately to the corresponding bidder, a unique symmetric equilibrium exists. Each agent bids her expected value conditioned on her opponent's signal being equal to her own,*

$$\begin{aligned} bid_L(L) &= \mathbb{E}[v|s_1 = L, s_2 = L] \\ &= P(G|s_1 = L, s_2 = L) \\ &= \frac{(1-p_1)(1-p_2)x}{(1-p_1)(1-p_2)x + q_1 q_2 (1-x)}, \end{aligned}$$

$$\begin{aligned} bid_H(H) &= \mathbb{E}[v|s_1 = H, s_2 = H] \\ &= P(G|s_1 = H, s_2 = H) \\ &= \frac{p_1 p_2 x}{p_1 p_2 x + (1-q_1)(1-q_2)(1-x)}. \end{aligned}$$

Proof. Assumption 2 (that bidders play only weakly undomainated strategies) restricts an agent with a *Low* signal to bid between $E[v|s_i = L, s_{-i} = L]$ and $E[v|s_i = L, s_{-i} = H]$, and one with a *High* signal to bid between $E[v|s_i = H, s_{-i} = L]$ and $E[v|s_i = H, s_{-i} = H]$. To see that the proposed strategy in proposition 1 is the only symmetric equilibrium, we begin by assuming that there exists a symmetric strategy that, when receiving signal L, the agent 1 bids x_1 and the agent 2 bids x_2, and when receiving signal H, Agent 1 bids y_1 and Agent 2 bids y_2. Suppose $x_1 \geq x_2$, then Agent 1 will be strictly better off by deviating to $E[v|s_i = L, s_{-i} = L]$ when receiving an L signal, since bidding x_1 could result in negative utility ($E[v|s_i = L, s_{-i} = L] - x_2$) if Agent 2 also receives an L signal. Similarly, if $y_1 \geq y_2$, Agent 2 has incentive to switch to $E[v|s_i = H, s_{-i} = H]$ when receiving an H signal to achieve higher expected utility. Thus, the equilibrium bids above constitute the only symmetric equilibrium. $\square$