1.  
   a. $\forall x \exists! y: B(x, y)$  
   b. $\forall x \exists y: (F(x) \land P(x)) \rightarrow M(x, y)$  
      
      “For all x where F(x) and P(x) are true, there exists some y that makes M(x,y) true” – this is precisely what we want to express.  
   c. $\exists m: \forall a: \exists f: P(m, f) \land Q(f, a)$  
      
      We begin by saying there exists a single man. Next, we want to say that given any airline, we can find a flight f on that airline that this man has taken, thus the quantifier order: “There exists a man $m$, such that for all airlines $a$ there is some flight $f$”

2.  
   a. $\forall b: Hummingbird(b) \rightarrow RichlyColored(b)$  
   b. $\forall b: \sim Large(b) \lor \sim LivesOnHoney(b)$ (or $\forall b: Large(b) \rightarrow \sim LivesOnHoney(b)$)  
   c. $\forall b: \sim LivesOnHoney(b) \rightarrow \sim RichlyColored(b)$  
   d. $\therefore Hummingbird(b) \rightarrow \sim Large(b)$  
      
      The conclusion is correctly drawn from the premises. Hummingbirds are given as richly colored, meaning that they must live on honey, or they would be dull colored. Because no large birds live on honey, and because all hummingbirds live on honey, no hummingbird can possibly be large. Therefore, all hummingbirds are small.

3.  
   a. For all $x$ and $y$, there exists some $z$ such that $x + y = z$ – **True**: given any two numbers, you can always find their sum $z$ by adding them together.  
   b. There exists a $z$ such that, for all $x$ and $y$, $x + y = z$ – **False**: given a single number, it is not the case that any two other numbers will add up to that number (ie, $3 = 2+1$, but $3 \neq 1+4$).
4.
   a. \( \forall x: \forall y: \neg P(x, y) \)
   b. \( \exists y: \forall x: \neg P(x, y) \)
   c. \( \forall y: (\neg Q(y) \lor \exists x: R(x, y)) \)
   d. \( \forall y: (\forall x: \neg R(x, y) \land \exists x: \neg S(x, y)) \)
   e. \( \forall y: (\exists x: \forall z: \neg R(x, y, z) \land \forall x: \exists z: \neg U(x, y, z)) \)

5.
   a. False
   
   \[ 2 > 1, \text{ but so is } 3. \]
   b. False
   
   \[ (-1)^2 = (1)^2 = 1 \]
   c. True
   
   \[ x = 3 \]
   d. False
   
   *There actually exists NO x where x = x + 1*

6.
   a. True
   
   If there exists a unique x such that P(x) is true, then surely there exists some x such that P(x) is true.
   
   b. False
   
   If every value of x makes P(x) true, then there does not exist only one value of x that makes P(x) true.
   
   c. True
   
   If there exists some x (even a unique one) that makes P(x) false, then surely it cannot be the case that all possible values of x make P(x) true.
Quiz

1. \[
\forall x: C(x) \rightarrow P(x) \\
\forall x: T(x) \rightarrow P(x) \\
\therefore \forall x: C(x) \rightarrow T(x)
\]
   Incorrect, implies is not the same as “if and only if.”

2. \[
\forall x: P(x) \rightarrow D(x) \\
\forall x: D(x) \rightarrow C(x) \\
\therefore \forall x: P(x) \rightarrow C(x)
\]
   Correct, by Universal Transitivity.

3. \[
\forall x: V(x) \rightarrow M(x) \\
\forall x: Q(x) \rightarrow V(x) \\
\therefore \forall x: Q(x) \rightarrow M(x)
\]
   Correct, by Universal Transitivity

4. \[
\forall x: S(x) \rightarrow G(x) \\
S(x) \\
\therefore G(x)
\]
   Correct, by modus ponens.

5. \[
\forall x: H(x) \rightarrow T(x) \\
\sim H(x) \\
\therefore \sim T(x)
\]
   Incorrect – The statement “H(x) \rightarrow T(x)” can, by simply looking at the truth table for implies, still be valid if H(x) is false and T(x) is also false.
6. \[
\forall x: E(x) \rightarrow \neg C(x)
\]
\[
\neg E(x)
\]
\[
\therefore C(x)
\]

**Incorrect** – If we replace E(x) with “H(x)” and “\neg C(x)” with “T(x)”, then this problem is identical to 5.

7. \[
\forall (x, y) \in \mathbb{Q}^2: (x + y) \in \mathbb{Q}
\]
\[
r + s \in \mathbb{Q}
\]
\[
\therefore r \in \mathbb{Q} \land s \in \mathbb{Q}
\]

**Incorrect** -- Converse error … A trivial counterexample is the case where \( r = \frac{\pi}{2} \) and \( s = -\frac{\pi}{2} \), then \( r + s = \frac{\pi}{2} + \left(-\frac{\pi}{2}\right) = 0 \) and \( 0 \in \mathbb{Q} \) but neither \( r \) nor \( s \) are rational.