Each individual problem is worth 5 points.

1. Represent the following sentences (in bold) in logic
   a. **Everyone has exactly one best friend.** Use B(x,y) as y is the best friend of x
   b. **If someone is a female and is a parent, then this person is someone’s mother.** Let F(x) and P(x) be “x is female” and “x is a parent”, and M(x,y) be “x is the mother of y”.
   c. **There is a man who has taken a flight on every airline in the world.** Let P(m, f) be “m has taken f” and Q(f, a) be “f is a flight on a”.

2. Represent the following argument, where the first 3 sentences are premises and the last is the conclusion. (think about it – does the conclusion follow the premises?)
   a. All hummingbirds are richly colored
   b. No large birds live on honey
   c. Birds that do not live on honey are dull in color
   d. Hummingbirds are small

3. Let Q(x, y, z) be the statement “x+y=z.” Translate the following sentences into English (3 points each) and then determine their truth values. Here $\forall$ means “for all” and $\exists$ means “there exists”.
   a. $\forall x \forall y \exists z Q(x,y,z)$
   b. $\exists z \forall x \forall y Q(x,y,z)$

4. Rewrite each of the following statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
   a. $\neg \exists y \exists x P(x,y)$
   b. $\neg \forall y \exists x P(x,y)$
   c. $\neg \exists y (Q(y) \land \forall x \neg R(x,y))$
   d. $\neg \exists y (\exists x R(x,y) \lor \exists x S(x,y))$
   e. $\neg \exists y (\exists x \exists z R(x,y,z) \lor \exists x \exists z U(x,y,z))$
5. The notation $\exists! x \ P(x)$ denotes “there exists a unique $x$ such that $P(x)$ is true”. If the universe of discourse is the set of integers, what are the truth values of the following?

   a. $\exists! x \ (x > 1)$

   b. $\exists! x \ (x^2 = 1)$

   c. $\exists! x \ (x + 3 = 2x)$

   d. $\exists! x \ (x = x + 1)$

6. What are the truth values of the following?

   a. $\forall x \ P(x) \rightarrow \exists x P(x)$

   b. $\forall x \ P(x) \rightarrow \exists! x P(x)$

   c. $\exists! x \neg P(x) \rightarrow \neg \forall x P(x)$