1. Use mathematical induction to prove that $2^n < n!$ for every positive integer $n > 3$.

Base Case: $n = 4$

$2^4 < 4!$

Inductive Step: Assume for $k$, $2^k < k!$

for $k+1$, $2^{k+1} = 2 \cdot 2^k$

$(k+1)! = (k+1) \cdot k!$

$2^k < (k+1) \cdot k!$

$2^{k+1} < (k+1)!$ since $k > 3$

2. Use mathematical induction to prove that $1 \cdot 2 + 2 \cdot 3 + \ldots + n \cdot (n+1) = n \cdot (n+1) \cdot (n+2)/3$

Base Case: $n = 1$

$(1 \cdot 2) / 3 = 2/3 = 2$

Inductive Step: $k+1 > k$

$1 \cdot 2 + 2 \cdot 3 + \ldots + k \cdot (k+1) + (k+1) \cdot (k+2) = \frac{(k+1)(k+2)(k+3)}{3}$

$\frac{k(k+1)(k+2)}{3} + \frac{(k+1)(k+2)}{3} = \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3} = \frac{(k+1)(k+2)(k+3)}{3}$

3. Use mathematical induction to prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

Base Case: $n = 12$, $12 = 4 + 4 + 4$

Inductive Step: Hypothesis: $n = k$, for $k \geq 12$, can be obtained from 4 + 5 cent stamps.

Two Cases:

If a 4 cent stamp is used to make up $k$ cents, replace it w/ a 5 cent stamp.

If no 4 cent stamp is used, then at least 3 5 cents stamps must be used for $n \geq 12$, so remove 3 5 cent stamps + replace w/ 4 cent stamps.
4. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

a.

b.

5.

a.

S_1 and S_2 each have eight elements

b.

S_1 \cup S_2 has 16 elements

c.

S_1 \cup S_2 = \emptyset(A)