A MinMax Example

<table>
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<tr>
<th></th>
<th>L</th>
<th>C</th>
<th>R</th>
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<tbody>
<tr>
<td>U</td>
<td>3, -3</td>
<td>-2, 2</td>
<td>2, -2</td>
</tr>
<tr>
<td>M</td>
<td>-1, 1</td>
<td>0, 0</td>
<td>4, -4</td>
</tr>
<tr>
<td>D</td>
<td>-4, 4</td>
<td>-3, 3</td>
<td>1, -1</td>
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“Pure strategy minmax” for Row player?
M means Column player can make at most 1

“Pure strategy minmax” for Column player?
C

(M, C) is not a Nash Equilibrium!
## Mixed Strategy MinMax

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Consider Column player playing \((1/3, 2/3, 0)\)

- R plays U: gets \(1 - 4/3 = -1/3\)
- R plays M: gets \(-1/3 + 0 = -1/3\)
- R plays D: gets \(-4/3 - 6/3 = -10/3\)

R is indifferent between U and M. Can guarantee herself a payoff of \(-1/3\) by mixing them \((1/6, 5/6)\)
Minimize $U_1^*$

subject to $\sum_{k \in A_2} u_1(a^j_1, a^k_2)s^k_2 \leq U_1^* \quad \forall j \in A_1$

$\sum_{k \in A_2} s^k_2 = 1$

$s^k_2 \geq 0$

Row player’s utility from any action must be either exactly the minmax value or less (in which case it will be played with 0 probability)

Constrain Column player’s strategy to be a probability distribution.
The Dual

Maximize $U_1^*$

subject to

$$\sum_{j \in A_1} u_1(a^j_1, a^k_2) s^j_1 \geq U_1^* \quad \forall k \in A_2$$

$$\sum_{j \in A_1} s^j_1 = 1$$

$$s^j_1 \geq 0$$

Row player’s utility under any action selected by Column player must be at least the maxmin value.

Constrain Row player’s strategy to be a probability distribution.

Computing Row player’s maxmin strategy!
Ben-Gurion’s Tri-lemma

(Based on James Stodder, “Strategic Voting and Coalitions: Condorcet’s Paradox and Ben-Gurion’s Tri-lemma” *Int. Rev. of Econ. Ed.* (2005))
Soviet era joke: God comes to the Soviet people and says: “I will give each of you a choice of three blessings in life, but you can only have two out of the three. You can be an honest person, you can be a smart person, or you can be a member of the Communist Party. If you are smart and honest, then you cannot be a communist. If you are a smart communist, then you cannot be honest. And if you are an honest communist, then obviously, you must not be very smart.”
Ben-Gurion’s “tri-lemma”

In November 1947 ... David Ben-Gurion, then the leader of the Zionist movement in Palestine ... did not shrink from clearly laying out the choice before the Jewish people ... Who were they? A nation of Jews living in all the land of Israel, but not democratic? A democratic nation in all the land of Israel, but not Jewish? Or a Jewish and democratic nation, but not in all the land of Israel? Instead of definitively choosing among these three options, Israel's two major political parties – Labor and Likud – spent the years 1967 to 1987 avoiding a choice ... not on paper, but in day-to-day reality.

(Friedman, 1989, pp. 253–4)
Your setting: Starting a business

G: Good works, H: Honesty, P: Profitability

**Left:** G > H > P

**Center:** P > G > H

**Right:** H > P > G
Options will be ranked.
Only two of three can be simultaneously picked.
The first one will be the primary goal of the company.

**First:** vote (and agree) on a finalist

**Second:** choose between the other two

**Third:** vote on top priority among the two finalists
Mechanics: Agenda Setting

• Each group will caucus together and pick a lead negotiator
• Lead negotiators will meet privately, in pairs, in sequence: 
  L+C, C+R, R+L
• Followed by another round of pairwise meetings (same sequence)
• Each group will submit a vote on one option (G, H, P) for finalist
• If no winner, repeat (with one round of pairwise meetings) until there is
Mechanics: Voting

Round 1: Each group caucuses and then picks one of the two remaining options to join the finalist

Round 2: Each group caucuses and then picks one of the two finalists as the priority
## Outcome values

**Left:** $G > H > P$

**Center:** $P > G > H$

**Right:** $H > P > G$

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<td>$G&gt;H&gt;P$</td>
<td>$3x3000+2x2000=13000$</td>
<td>$3x2000+2x1000=8000$</td>
<td>$3x1000+2x3000=9000$</td>
</tr>
<tr>
<td>$H&gt;G&gt;P$</td>
<td>$3x2000+2x3000=12000$</td>
<td>$3x1000+2x2000=7000$</td>
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