Note Title 4/16/2009

Scaling

parallel system $\triangleq$ parallel alg. + parallel arch.

T_p execution time on p processors

T_1 serial execution time

traditional speedup (fixed problem size)

$$S = \frac{T_1}{T_p}$$

efficiency

$$E = \frac{S}{p} = \frac{T_1}{pT_p}$$

Scaled speedup (prob size increases linearly w/ p)

$$S = \frac{pT_1}{T_p}$$ small prob on 1 proc

$$E = \frac{pT_1}{pT_p} = \frac{T_1}{T_p} \quad \text{scaled efficiency}$$

add n numbers on p proc

3	7	11	15
2	6	10	14
1	5	9	13
0	4	8	12

$\sum_0^3 \quad \sum_4^7 \quad \sum_8^{11} \quad \sum_{12}^{15}$

$\sum_0^7 \quad \sum_8^{16}$

$\sum_0^{15}$

n/p time

$\log p$ msg adds

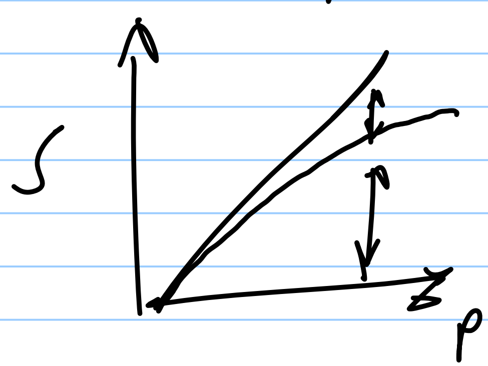
$$T_p = (n/p + 2\log p) \times t_c$$

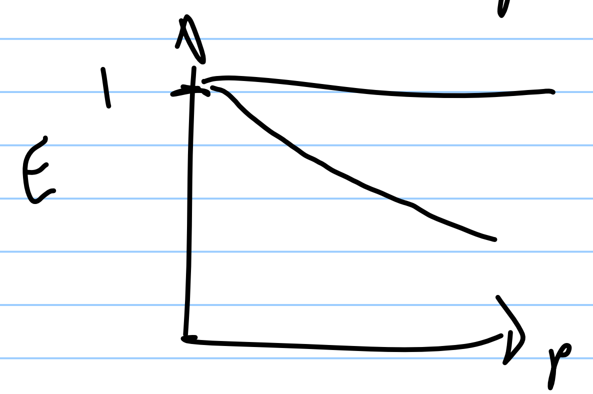
$$T_1 = n \times t_c$$

Speedup = p is called linear

$$S = \frac{n}{n/p + 2\log p}$$

$$E = \frac{n}{n + 2p\log p}$$





fixed n

scaling both n and p

can I keep E fixed? If so, scalable

how fast must we scale work wrt p to fix E?

$\Rightarrow$ isoefficiency function

$$E = \frac{n}{n + 2p\log p}$$

$\epsilon n + 2\epsilon p \log p = n$

$n - \epsilon n = 2\epsilon p \log p$

$n = \frac{2\epsilon}{1-\epsilon} p \log p$

$\Theta(p \log p)$

$\Theta(p)$ is best

Amdahl's Law

upper bound on speedup, fixed size prob

f_s : fraction of serial execution time that stays serial

f_p - " , " " " " is parallelized

$$T_p \geq T_1 \left(f_s + \frac{f_p}{p} \right)$$

$$S = \frac{T_1}{T_p} \leq \frac{T_1}{T_1 (f_s + f_p/p)} \leq \frac{1}{f_s + f_p/p}$$

T_1 $\quad \boxed{\; f_s \;\big|\; f_p \;}$

T_p $\quad \boxed{\; f_s \;\big|\; \frac{f_p}{p} \;}$

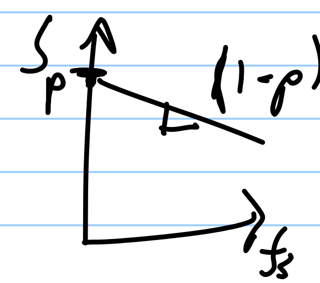
$$\lim_{p \to \infty} S = \frac{1}{f_s}$$

Gustafson's Law

scaled problem

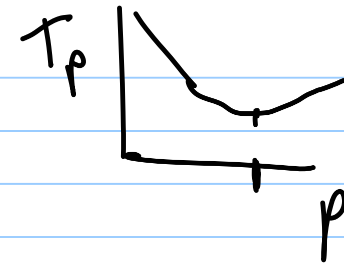
$$\text{scaled speedup} \;\overset{\rightarrow}{=}\; \frac{\text{hypothetical scaled prob on 1 proc.}}{T_p}$$

$$S \le \frac{f_s + P f_p}{f_s + f_p} \qquad \text{sub. } f_p = 1 - f_s$$



$$S \le f_s + P(1-f_s) = f_s + P - Pf_s$$
$$\le P + (1-P) f_s$$

$$T_p = \frac{n}{p} + 2\log p$$



$$\frac{d}{dp}(T_p) = -\frac{n}{p^2} + 2\frac{1}{p} = 0$$

$$-\frac{n}{p} + 2 = 0$$

$$p = \frac{n}{2}$$

$$T_p = \frac{n}{p} + 1000 \log p$$

$$p = \frac{n}{1000}$$

$$p = \frac{n}{r}$$