Scaling

Note Title 4/16/2009

parallel system $\triangleq$ parallel alg. + parallel arch.

T_p execution time on p processors

T_1 serial execution time

traditional speedup (fixed problem size)

$$S = \frac{T_1}{T_p}$$

efficiency

$$E = \frac{S}{p} = \frac{T_1}{pT_p}$$

Scaled speedup (prob size increases linearly w/ p)

$$S = \frac{pT_1}{T_p}$$ small prob on 1 proc

$$E = \frac{pT_1}{pT_p} = \frac{T_1}{T_p}$$ scaled efficiency

add n numbers on p proc

3	7	11	15
2	6	10	14
1	5	9	13
0	4	8	12

$\Sigma_0^3 \quad \Sigma_4^7 \quad \Sigma_8^{11} \quad \Sigma_{12}^{15}$

$\Sigma_0^7 \quad \Sigma_8^{15}$

Σ_0^{15}

n/p time

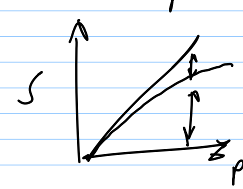
$\log p$ msg adds

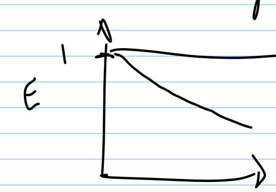
$$T_p = (n/p + 2\log p) \times t_c$$

$$T_1 = n \times t_c$$

Speedup $= p$ is called linear

$$S = \frac{n}{\frac{n}{p} + 2\log p} \qquad E = \frac{n}{n + 2p\log p}$$





fixed n

scaling both n and p

can I keep E fixed? If so, scalable

how fast must we scale work wrt p to fix E?

$\Rightarrow$ isoefficiency function

$$E = \frac{n}{n + 2p\log p}$$

$$En + 2Ep\log p = n$$

$$n - En = 2Ep\log p$$

$$n = \frac{2E}{1-E}\, p\log p$$

$$\Theta(p\log p)$$

$\Theta(p)$ is best

Amdahl's Law

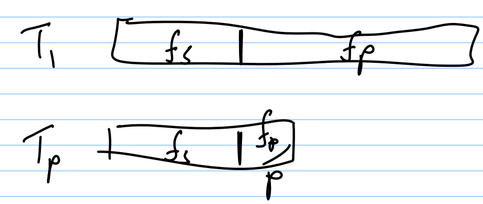
upper bound on speedup, fixed size prob

f_s : fraction of serial execution time that stays serial

f_p : " " " " " is parallelized

$$T_p \geq T_1\left(f_s + \frac{f_p}{p}\right)$$

$$S = \frac{T_1}{T_p} \leq \frac{T_1}{T_1(f_s + f_p/p)} \leq \frac{1}{f_s + f_p/p}$$



$$\lim_{p\to\infty} S = \frac{1}{f_s}$$

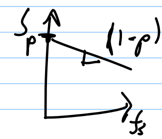
Gustafson's Law

scaled problem

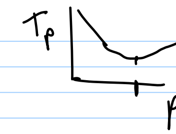
$$\text{scaled speedup} = \frac{\text{hypothetical scaled prob on 1 proc.}}{T_p}$$

$$S \le \frac{f_s + p f_p}{f_s + f_p} \qquad \text{sub. } f_p = 1 - f_s$$

$$S \le f_s + p(1 - f_s) = f_s + p - p f_s$$
$$\le p + (1-p) f_s$$



$$T_p = \frac{n}{p} + 2 \log p$$



$$\frac{d}{dp}(T_p) = -\frac{n}{p^2} + 2\frac{1}{p} = 0$$

$$-\frac{n}{p} + 2 = 0$$

$$p = \frac{n}{2}$$

$$T_p = \frac{n}{p} + 1000 \log p$$

$$p = \frac{n}{1000}$$

$$p = \frac{n}{r}$$