Price Evolution in a Continuous Double Auction Prediction Market With a Scoring-Rule Based Market Maker

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Introduction

- Prediction Market: online platform for trading a financial instrument tied to some future uncertain event.
  - Elicits diffuse private information.
  - Produces collective forecast.
- Popular methods of implementation:
  - Continuous Double Auction (CDA)
  - Logarithmic Market Scoring Rule (LMSR)

Goal: To extend an existing way of integrating CDA with LMSR market maker (MM) such that MM can be implemented as just another trader (requiring no “special access” to incoming orders and books), and trading is more intuitive to traders.

Trader Model (extended ZI)

Each simulation, at \( t = 0 \):
- True probability of occurrence of event \( p_{\text{mm}} \sim U[0,1] \).
- Trader \( i \) observes outcome of N-coin-flips with prior heads \( p_{\text{mm}} \) and hence estimates valuation \( v \).
- Higher \( N \) — higher precision in belief.
At each \( t \), one trader:
- Picked uniformly at random.
- Buy/sell with equal probability (except for pureLMSR).
- Ask: \( v \sim U[0,v] \) if seller, \( v \sim U[v,1] \) if buyer, and quantity \( q \sim \text{exp}(4) \).

Information Aggregation Properties

- **Convergence time (ConvTime)**: \# time-steps for “market price” (instantaneous price for LMSR, mid-point of bid-ask spread for others) to enter band of size \( \varepsilon = 0.05 \) around \( p_{\text{mm}} \).
- **Volatility measures**: Root-mean-squared deviation of market price \( p_{\text{mm}} \) from over entire simulation (RMSD) and over “equilibrium period”, i.e., \( t \geq \text{ConvTime} \) (RMSD).

Market Quality Properties

- **Bid-ask spread**: Time-average of (best ask – best bid) over entire simulation (Spread) and over equilibrium period (Spread \( * \)).
- **Bid-price consistency** (TraderSurplus): If trader with valuation \( v \) places buy or sell order of which quantity \( q \) executes at price \( p_{\text{mm}} \), then individual surplus is \( q(v - p_{\text{mm}}) \) or \( q(p_{\text{mm}} - v) \) respectively. Sum of these surpluses = TraderSurplus.
- **MMLoss**: Total loss incurred by market maker computed like (negative of) individual trader surplus, \( v \) replaced with \( p_{\text{mm}} \).

Market Microstructures Compared

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<th>pureCDA</th>
<th>pureLMSR</th>
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<td>Two order books — outstanding buy and sell orders</td>
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<td>Orders prioritized by limit price and arrival time</td>
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</table>
| If highest bid \( b_{\text{max}} \) \( \geq \) lowest ask \( a_{\text{min}} \):
  - trade smaller quantity at limit price of earlier order
  - update books; continue till \( b_{\text{max}} < a_{\text{min}} \) |

For buy order \( q, p \): similar for sell order:
- If \( p \leq p_{\text{mm}} \) push order on to book.
- Else if \( q \) not enough to drive \( p_{\text{mm}} \) beyond \( \min\{p_{\text{mm}}, a_{\text{min}}\} \), execute order fully with LMSR.
- Else if \( p_{\text{mm}} < p \leq a_{\text{min}} \), execute order partially with LMSR till \( p_{\text{mm}} \) reaches \( p \); place residual order on book.
- Else LMSR sells till \( p_{\text{mm}} \) hits \( a_{\text{min}} \); remaining order executes against best booked ask.
- If top level of ask book is exhausted but not incoming order, invoke LMSR again — recur till order is finished or new \( a_{\text{min}} > p_{\text{mm}} \). |

Market maker takes one side of every trade.
- “State” \( q(t) \) — total quantity of outstanding shares
- Instantaneous price \( p_{\text{mm}} = \frac{q(t) \cdot B}{B + q(t) + Q} \) \( B > 0 \) |
- If trader buys \( Q \) (\( Q < 0 \) for selling), update \( a_{\text{min}} \) to \( (q(t) + Q) / (Q + Q) \); trader’s cost is integral of \( p_{\text{mm}} \) from \( q_{\text{mm}} \) to \( (q_{\text{mm}} + Q) \).

Integrating CDA with LMSR (INT)

Whenever best bid or ask changes,\( t \):
1. If \( p_{\text{mm}} \geq b_{\text{max}} \) MM generates limit sell order that, if fully executed immediately, would take \( p_{\text{mm}} \) to \( a_{\text{min}} \).
2. If \( p_{\text{mm}} \leq a_{\text{min}} \), it generates limit buy order that would take \( p_{\text{mm}} \) to \( b_{\text{max}} \).
3. Otherwise, it generates both orders.
  - Reduces all earlier orders with new order(s) only if it does not immediately cross books.
  - CDA mechanism proceeds.

Evaluation

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<th>PAR (Hanson, 2003)b</th>
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Implementation details

3 sets of 1000 simulations each for \( N = 20,40,100 \), \# time-steps per simulation = 500, \( B = 100, A = 20 \).
- \( \text{Vol} \) — quantity absorbed by buyers and market maker (if present) = quantity supplied by sellers and market maker (if present).
- \( \text{Vol}^* \) — quantity absorbed by buyers (from sellers and market maker) + quantity supplied by sellers (to buyers and market maker).

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Acknowledgments

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