CSE 316A: Homework 2

Due on October 12, 2015
Total: 100 points + 20 bonus points

Notes

- There are 6 problems on 4 pages below, worth a total of 120 points. However, this homework will be graded out of 100 points, and any additional points you gain out of the 20 bonus points will count towards your overall homework score (40% of your total score for the course).

- **Homework (typed or hand-written) must be handed in to me individually at the beginning of lecture on Monday, October 12, between 2:30 pm and 2:45 pm.**

- **You must write / type all your answers on your own.**

- Before you prepare your submission, you are free to discuss any problem with other students in this course and / or use resources other than the textbook and lecture notes. But you must clearly mention the students with whom you discussed the homework as well as any outside resources you used on your submission.

- Please review the academic integrity policy and grade appeal policy specified in the official course syllabus at http://www.cse.wustl.edu/~mithunchakraborty/SyllabusCSE316A.pdf.

- My office hours are Tuesdays 2:00 pm to 4:00 pm in Bryan 503.

Problems

1. Consider two Internet Service Providers (ISPs), $A$ and $B$, with networks as depicted in Figure 1. There are two shared nodes, $C$ and $S$, called peer points, where the two ISPs can exchange traffic. Assume that each link in each network can carry any amount of traffic in either direction. Moreover, when a packet is routed from its source to its target, an ISP incurs a cost proportional to the number of links along which the packet travels within its own network.

Every ISP, in making routing decisions, does whatever is in its power to reduce its own cost. However, once a packet destined for a target node within a network enters the network, the corresponding ISP must route it to the target.

Suppose that two packets of data originate at source nodes $s_1$ and $s_2$ within the networks of ISPs $A$ and $B$ (one at each) at the same time, and must be sent to target nodes $t_1$ and $t_2$
within the networks of $B$ and $A$ respectively, as in Figure 1.

Figure 1: Problem 1.

(a) What are the different ways in which each ISP can route the packet originating within its own network towards its target? (5 points)

(b) Model the above situation as a game (specify the players and their available actions, and construct the payoff matrices). Predict an outcome for this game in terms of any solution concept discussed in class (whichever is / are applicable), and justify your answer. What is the best possible outcome from ISP $A$’s point of view? From ISP $B$’s point of view? (15 points)

2. Recall the basic description of the prisoners’ dilemma game as presented in class:

Two individuals (call them Karkaroff and Malfoy), suspected of being involved in a crime together, are being interrogated in separate rooms so that they cannot communicate with each other, and each is given the same two choices by law enforcement personnel: He can either stay silent (choice 1) or confess to his involvement and snitch on the other person (choice 2).

If both stay silent, each is sentenced to 3 years in prison (for want of clinching evidence). If one suspect confesses and the other stays silent, the one who confesses gets only 1 year (for cooperating with law enforcement), and the other has to serve 10 years. If both confess (and snitch), each gets 5 years in prison.

For each of the following modified versions of the above game (in each of which all the above information still holds), construct a bi-matrix representation that you think is an acceptable model for the situation, and make a reasonable prediction for the outcome: State your solution concept clearly, and give a brief explanation of your answer.

(a) Suppose that both players are completely selfless, and each cares only about reducing the other player’s prison sentence with no regard to his own fate. (10 points)
(b) Suppose that each player “feels the other’s pain” in addition to his own, i.e. if Karkaroff
serves $x$ years in prison and Malfoy $y$ years, each suffers a cost of $(x + y)$. \(10 \text{ points}\)

(c) Suppose that Malfoy has connections to important people in law enforcement so that
if both suspects stay silent, Malfoy gets off scot-free, i.e. serves no prison time at all,
while Karkaroff gets 2 years. Each suspect cares about reducing his own prison sentence
selfishly. \(10 \text{ points}\)

[**Food for thought, no response required:** For versions (a) and (b), can our players still
be called “selfish-rational” according to the technical definition provided in class?]

3. Consider the game described in Table 1, and write down the inequalities that the entries in
the payoff matrices must satisfy such that \((U, L)\) is a strict (pure-strategy) Nash equilibrium
of this game but not a dominant strategy equilibrium (even in a weak sense). Under these
conditions (i.e. if the inequalities that you just wrote down are true), is it possible for \((U, L)\)
to be the unique Nash equilibrium of the game? Explain your answers. \(20 \text{ points}\)

<table>
<thead>
<tr>
<th>Player B</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U</strong></td>
<td>$a_{UL}$, $b_{UL}$</td>
<td>$a_{UR}$, $b_{UR}$</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>$a_{DL}$, $b_{DL}$</td>
<td>$a_{DR}$, $b_{DR}$</td>
</tr>
</tbody>
</table>

Table 1: Payoff matrices for the two-player game in Problem 3.

4. You and a friend are playing a game whose rules are as follows. Each of you is asked to
pick any number from the list \{1, 2, 3, 4, 5, 6, 8, 9, 100\} without any knowledge of the other’s
choice, and then submit your choices to a gamemaster separately.\(^1\) If both of you choose the
same number, each of you gets the same payoff, say 10; if you choose different numbers, you
both get zero payoffs.

(a) What are the pure-strategy Nash equilibria of the game? Explain briefly without con-
structing payoff matrices explicitly. \(10 \text{ points}\)

(b) \((\text{Bonus})\) First, convince yourself that the above equilibria are not different from each
other in terms of the payoffs they give to each of the players (no response required).
Regardless of this observation, would you say that one of these Nash equilibria is, in
some sense, a more reasonable prediction for the outcome of this game than the others.
Justify your response. For doing so, you might need to do some additional reading / \(10 \text{ bonus points}\)
research; feel free to do so, but please cite your sources.

5. Recall the Golden Balls game described in class (see Table 2 below). Find all Nash equilibria
for this game. \(20 \text{ points}\)

\(^1\) Also assume that neither of you had ever anticipated participating in a game of this kind, and hence have never
talked about how to act in a situation like this.
6. (Bonus) Recall the pollution game as described in class: There are $n = 4$ players, each with the option to be a good neighbor or a bad neighbor. If a player is a good neighbor, it suffers a cost of 3 itself but does not cause any of the other players to suffer any cost. But, if a player is a bad neighbor, it makes every player (including itself) incur a cost of 1. A player’s payoff for any outcome is just the negative of its overall cost.

Argue that it is a strictly dominant strategy for any player to be a bad neighbor not just for $n = 4$ (as demonstrated in class) but for any integer $n \geq 4$. (10 bonus points)