Examples of Schedulability Analysis

1.

a) Is the following set of periodic tasks schedulable under Rate Monotonic (RM)?
b) How about under Deadline Monotonic (DM)?
c) How about under Earliest Deadline First (EDF)?

$C_i, D_i,$ and $P_i$ denote the worst-case execution time, relative deadline, and period of task $T_i$, respectively.

<table>
<thead>
<tr>
<th></th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$D_i$</td>
<td>5</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>$P_i$</td>
<td>5</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

Answer:

a) The RM utilization bound test is NOT applicable because deadline $\neq$ period in the task sets. We apply response time analysis test. The (decreasing) priority order under RM is $T_1, T_3, T_2$.

$T_1$:
1) Since $T_1$ has the highest (fixed) priority, $R = C_1 = 1$. Since $R \leq D_1$, $T_1$ is schedulable.

$T_3$:
1) $R = I + C_3 = 0 + 2 = 2, I = \left[ \frac{R}{P_1} \right] * C_1 = \left[ \frac{2}{5} \right] * 1 = 1$.
   Since $R \leq D_3$ and $I + C_3 = 3 > R$, continue.
2) $R = 3, I = \left[ \frac{R}{P_1} \right] * C_1 = \left[ \frac{3}{5} \right] * 1 = 1$.
   Since $R = I + C_3 = 3$, the worst-case response time of $T_3$ is 3.
   Since $R < D_3 = 4$, $T_3$ is schedulable.

$T_2$:
1) $R = I + C_2 = 0 + 4 = 4, I = \left[ \frac{R}{P_1} \right] * C_1 + \left[ \frac{R}{P_3} \right] * C_3 = \left[ \frac{4}{5} \right] * 1 + \left[ \frac{4}{6} \right] * 2 = 3$.
   Since $R < D_2$ and $I + C_2 = 7 > R$, continue.
2) $R = 7, I = \left[ \frac{R}{P_1} \right] * C_1 + \left[ \frac{R}{P_3} \right] * C_3 = \left[ \frac{7}{5} \right] * 1 + \left[ \frac{7}{6} \right] * 2 = 6$.
   Since $R < D_2$ and $I + C_2 = 10 > R$, continue.
3) $R = 10$.
   Since $R > D_2$, $T_2$ is NOT schedulable.

Hence the task set is NOT schedulable under RM.

b) We apply response time analysis test. The (decreasing) priority order under DM is $T_3, T_1, T_2$. Note $T_3$ has a higher priority than $T_1$ under DM.

$T_3$: Since $T_3$ has the highest priority, $R = C_3 = 2$. Since $R \leq D_3$, $T_3$ is schedulable.
T1:
1) \( R = I + C_1 = 0 + 1 = 1 \), \( I = \left\lceil \frac{R}{P_3} \right\rceil \cdot C_3 = \left\lceil \frac{1}{6} \right\rceil \cdot 2 = 2 \).
   Since \( R \leq D_1 \) and \( I + C_1 = 3 > R \), continue.
2) \( R = 3 \), \( I = \left\lceil \frac{R}{P_3} \right\rceil \cdot C_3 = \left\lceil \frac{3}{6} \right\rceil \cdot 2 = 2 \).
   Since \( R = I + C_1 = 3 \), the worst-case response time of T1 is 3.
   Since \( R < D_1 = 5 \), T1 is schedulable.

T2:
1) \( R = I + C_2 = 0 + 4 = 4 \), \( I = \left\lceil \frac{R}{P_1} \right\rceil \cdot C_1 + \left\lceil \frac{R}{P_3} \right\rceil \cdot C_3 = \left\lceil \frac{4}{5} \right\rceil \cdot 1 + \left\lceil \frac{4}{6} \right\rceil \cdot 2 = 3 \).
   Since \( R < D_2 \) and \( I + C_2 = 7 > R \), continue.
2) \( R = 7 \), \( I = \left\lceil \frac{R}{P_1} \right\rceil \cdot C_1 + \left\lceil \frac{R}{P_3} \right\rceil \cdot C_3 = \left\lceil \frac{7}{5} \right\rceil \cdot 1 + \left\lceil \frac{7}{6} \right\rceil \cdot 2 = 6 \).
   Since \( R < D_2 \) and \( I + C_2 = 10 > R \), continue.
3) \( R = 10 \).
   Since \( R > D_2 \), T2 is NOT schedulable.
   Hence the task set is NOT schedulable under DM.

c) The utilization bound test is NOT applicable to this case because the deadlines do not equal to corresponding periods. Instead, the processor demand test must be applied.

Step 1: Compute busy period
1) \( L = C_1 + C_2 + C_3 = 7 \);
   \( L' = W(L) = \left\lceil \frac{L}{T_1} \right\rceil \cdot C_1 + \left\lceil \frac{L}{T_3} \right\rceil \cdot C_3 + \left\lceil \frac{L}{T_2} \right\rceil \cdot C_2 = \left\lceil \frac{7}{5} \right\rceil \cdot 1 + \left\lceil \frac{7}{6} \right\rceil \cdot 2 + \left\lceil \frac{7}{9} \right\rceil \cdot 4 = 10 \);
   \( H = \text{lcm}(T_1, T_2, T_3) = 90 \).
   Since \( L' \neq L \) and \( L' \leq H \), continue.
2) \( L = L' = 10 \);
   \( L' = W(L) = \left\lceil \frac{10}{5} \right\rceil \cdot 1 + \left\lceil \frac{10}{6} \right\rceil \cdot 2 + \left\lceil \frac{10}{9} \right\rceil \cdot 4 = 14 \);
3) \( L = L' = 14 \);
   \( L' = W(L) = \left\lceil \frac{14}{5} \right\rceil \cdot 1 + \left\lceil \frac{14}{6} \right\rceil \cdot 2 + \left\lceil \frac{14}{9} \right\rceil \cdot 4 = 17 \);
4) \( L = L' = 17 \);
   \( L' = W(L) = \left\lceil \frac{17}{5} \right\rceil \cdot 1 + \left\lceil \frac{17}{6} \right\rceil \cdot 2 + \left\lceil \frac{17}{9} \right\rceil \cdot 4 = 18 \);
5) \( L = L' = 18 \);
   \( L' = W(L) = \left\lceil \frac{18}{5} \right\rceil \cdot 1 + \left\lceil \frac{18}{6} \right\rceil \cdot 2 + \left\lceil \frac{18}{9} \right\rceil \cdot 4 = 18 \);
   Since \( L' = L \) and \( L' \leq H \), the busy period \( Bp = L = 18 \).

Step 2: \( D = \{4, 5, 8, 10, 15, 16, 17\} \)

\[ L = 4 \geq \left\lceil \frac{(L - D_1) / P_1 + 1} \right\rceil \cdot C_1 + \left\lceil \frac{(L - D_3) / P_3 + 1} \right\rceil \cdot C_3 + \left\lceil \frac{(L - D_2) / P_2 + 1} \right\rceil \cdot C_2 = \left\lceil \frac{(4 - 5) / 5 + 1} \right\rceil \cdot 1 + \left\lceil \frac{(4 - 4) / 6 + 1} \right\rceil \cdot 2 + \left\lceil \frac{(4 - 8) / 9 + 1} \right\rceil \cdot 4 = 2 \]

\[ L = 5 \geq \left\lceil \frac{(5 - 5) / 5 + 1} \right\rceil \cdot 1 + \left\lceil \frac{(5 - 4) / 6 + 1} \right\rceil \cdot 2 + \left\lceil \frac{(5 - 8) / 9 + 1} \right\rceil \cdot 4 = 1 + 2 = 3 \]

\[ L = 8 \geq \left\lceil \frac{(8 - 5) / 5 + 1} \right\rceil \cdot 1 + \left\lceil \frac{(8 - 4) / 6 + 1} \right\rceil \cdot 2 + \left\lceil \frac{(8 - 8) / 9 + 1} \right\rceil \cdot 4 = 1 + 2 + 4 = 7 \]
\[ L = 10 \geq (\left\lfloor \frac{10-5}{5} \right\rfloor +1) \times 1 + (\left\lfloor \frac{10-4}{6} \right\rfloor +1) \times 2 + (\left\lfloor \frac{10-8}{9} \right\rfloor +1) \times 4 = 2 + 4 + 4 = 10 \]

\[ L = 15 \geq (\left\lfloor \frac{15-5}{5} \right\rfloor +1) \times 1 + (\left\lfloor \frac{15-4}{6} \right\rfloor +1) \times 2 + (\left\lfloor \frac{15-8}{9} \right\rfloor +1) \times 4 = 3 + 4 + 4 = 11 \]

\[ L = 16 \geq (\left\lfloor \frac{16-5}{5} \right\rfloor +1) \times 1 + (\left\lfloor \frac{16-4}{6} \right\rfloor +1) \times 2 + (\left\lfloor \frac{16-8}{9} \right\rfloor +1) \times 4 = 3 + 6 + 4 = 13 \]

\[ L = 17 \geq (\left\lfloor \frac{17-5}{5} \right\rfloor +1) \times 1 + (\left\lfloor \frac{17-4}{6} \right\rfloor +1) \times 2 + (\left\lfloor \frac{17-8}{9} \right\rfloor +1) \times 4 = 3 + 6 + 8 = 17 \]

Hence all tasks are schedulable under EDF.

2. Priority Inheritance
Consider the following three periodic tasks T1, T2, and T3 (having decreasing priority) scheduled under RM. All the critical sections are shown below. Access to semaphores (s1, s2, and s3) is controlled by the Priority Inheritance Protocol. The worst-case execution times of functions g(), h(), and m() are 2, 3, and 4 respectively. The execution times of P() and V() are assumed to be 0. Please compute the maximum blocking time of each task.

\[ \text{T1:} \]
\[ \ldots \]
\[ \text{wait(s1);} \]
\[ g(); \]
\[ \text{signal(s1);} \]
\[ \ldots \]
\[ \text{wait(s2);} \]
\[ g(); \]
\[ \text{signal(s2);} \]
\[ \ldots \]
\[ \text{wait(s3);} \]
\[ h(); \]
\[ \text{signal(s3);} \]

\[ \text{T2:} \]
\[ \ldots \]
\[ \text{wait(s1);} \]
\[ h(); \]
\[ \text{signal(s1);} \]
\[ \ldots \]
\[ \text{wait(s3);} \]
\[ m(); \]
\[ \text{signal(s3);} \]

\[ \text{T3:} \]
\[ \ldots \]
\[ \text{wait(s2);} \]
\[ h(); \]
\[ \text{signal(s2);} \]
\[ \ldots \]
\[ \text{wait(s3);} \]
\[ m(); \]
\[ \text{signal(s3);} \]

\[ \text{wait(s1);} \]
\[ m(); \]
\[ \text{signal(s1);} \]

Answer
The ceiling of each semaphore is shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>S1(T1)</th>
<th>S2(T1)</th>
<th>S3(T1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>T2</td>
<td>3</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>T3</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

T1: Bl=4+4=8; Bs=4+3+4=11. B1=min(Bl,Bs)=8.
T2: Bl=4=4; Bs=4+3+4=11. B2=min(Bl,Bs)=4.
T3: B3 = 0.