Video Traffic Modeling
Using Seasonal ARIMA Models

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These slides are available on-line at:
http://www.cse.wustl.edu/~jain/wimax/video82.htm
Overview

1. Video Compression overview
2. Seasonal ARIMA Model
3. Model for all frames combined
4. Models for I, P, B frames
Goals

- One approach to model video traffic is to just determine the distribution of the frame sizes.
- This assumes that the traffic is white noise and there is no predictability.
- If there is predictability, it can be used for resource management.
- Predictability $\Rightarrow$ Correlation in frame sizes.
- In video traffic, there is significant correlation. If a particular scene takes place in a complex background $\Rightarrow$ Large frame sizes through out the scene.
- Similarly, if the scene is simple, the frame sizes will remain low through out the scene.
Group of Pictures

A Typical Group Of Pictures (GOP)

Transmission Order of a GOP
MPEG Encoding

- Video Scene is divided into group of pictures (GOP)
- The default GOP size for NTSC: 15
- GOP consists of:
  - **I (Intracoded) frames**: Each frame of video is still discrete and can be accessed as a unique point
  - **P (Predicted) frames**: P frames are predicted based on prior I or P frames
  - **B (bi-directional predicted) frames**: B frames are coded based on a forward prediction from a previous I or P frame, as well as a backward prediction from a succeeding I or P frame
Using ffmpeg library, we encoded 1:24 minutes movie clip
- At 350kbps
- QVGA: 320x240
- GOP size of 15

GOP sequence is:
- IBBPBBPBBPBBPBBPBB
Video Frames
Video Frames: A Closer Look

Observation: Every 15th frame is a large (I) frame.
Observation: I, P, B frames have very different levels.
Auto-Regressive Models

- **AR(1) Model:**
  \[ y(t) = a(1) y(t-1) + w(t) \]

- **AR(p) Model:**
  \[ y(t) = a(1) y(t-1) + a(2) y(t-2) + \ldots + a(p) y(t-p) + w(t) \]

- **AR(0) Model:** \[ y(t) = w(t) \Rightarrow \text{White noise} \]

- **Auto-Correlation Function (ACF):**
  \[ AC(k) = \frac{\text{cov}(Y(t), y(t-k))}{\text{Var}(y(t))} \]

- **Partial Auto-Correlation Function (PACF):**
  \[ \text{PACF cuts off at } p \text{ for AR}(p) \]
Moving Average Models

- **MA(1) Model:**
  \[ y(t) = w(t) + b(1) w(t-1) \]

- **MA(q) Model:**
  \[ y(t) = w(t) + b(1) w(t-1) + b(2)w(t-2) + \ldots + b(q)w(t-q) \]

- **MA(0) Model:**
  \[ Y(t) = w(t) \Rightarrow \text{White noise} \]

ACF cuts off at q for MA(q)

\[ b_0 \quad b_1 \quad b_2 \quad b_3 \]

Lag k

PACF(k)
ARIMA Models

- **ARMA(p,q) Model:**
  \[ y(t) + a(1)y(t-1) + \ldots + a(p)y(p) = w(t) + b(1)w(t-1) + \ldots + b(q)w(t-q) \]

- **Using Backward operator:**
  \[ Dy(t) = y(t-1) \]
  \[ \{1+a(1)D+a(2)D^2+\ldots+a(p)D^p\}y(p) = \{1+b(1)D+\ldots+b(q)D^q\}w(t) \]
  \[ A(D)y(p) = B(D)w(t) \]

- **Auto-regressive Integrated Moving Average Model:**
  **ARIMA(p,d,q)**
  \[ A(D)(1-D)^dy(p) = B(D)w(t) \]

- **Example: ARIMA(1,1,1) Model**
  \[ (1+a_1D)(1-D)y(t) = (1+b_1D)w(t) \]
  \[ (1+a_1D-D-a_1D^2)y(t) = (1+b_1D)w(t) \]
  \[ y(t) - (1-a_1)y(t-1) - a_1y(t-2) = w(t) + b_1w(t-1) \]
**Seasonal ARIMA Model**

- Period of 12 ⇒ Model \( y(t)-y(t-12) \) as a ARIMA\((p,d,q)\) series
- ACF will show spikes at seasonal period \( s \)
- Seasonal ARIMA\((p,d,q)\) \((P,R,Q)s\) model:
  \[
  A(D)F(D^s)(1-D^s)Ry(t)=B(D) G(D^s)w(t)
  \]
- Example: \((1,0,0)x(0,1,0)_{12}\) Model
  \[
  (1+a_1D)(1-D^{12})y(t)=w(t)
  \]
  \[
  y(t)-y(t-12)-a(1)\{y(t-1)-y(t-13)\}=w(t)
  \]
### Interpreting ACF and PACF

<table>
<thead>
<tr>
<th></th>
<th>AR($p$)</th>
<th>MA($q$)</th>
<th>ARIMA($p,q$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACF</td>
<td>Trails off</td>
<td>Cuts off at lag $q$</td>
<td>Trails off</td>
</tr>
<tr>
<td>PACF</td>
<td>Cuts off at lag $p$</td>
<td>Trails off</td>
<td>Trails off [1]</td>
</tr>
</tbody>
</table>

Trails off at lag = 1
OR cuts off at lag = 2

[1]: Time Series Analysis and Its Applications: With R Examples
ACF (Auto-Correlation Function) graph shows many trends and show how much video frames are correlated.
A closer look at the ACF graph shows a strong continual correlation every 15 lag ⇒ GOP size
ACF graph after second level of differentiation with seasonal part of 15
PACF (Partial Auto-Correlation Function) shows another decaying trend around lags 15,30,45. Also around 14,29,44.
Within the same seasonal part: cuts off/trails at $h=1$, (ignore at lag 14)
Across seasonal parts: trails at $h=1$
Traffic Modeling – All Frames 6

- Within the same seasonal part: trails at h=1
- Across seasonal parts: trails at h=1
The chosen seasonal ARIMA model is:

$$(1,1,1) \times (1,1,1)_{15}$$

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar (1)</th>
<th>ma(1)</th>
<th>sar(1)</th>
<th>sma(1)</th>
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<tbody>
<tr>
<td></td>
<td>0.2675</td>
<td>-1.000</td>
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<td>-0.6723</td>
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<tr>
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<td>0.0018</td>
<td>0.0392</td>
<td>0.0298</td>
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</table>
Results 1

Standardized Residuals

ACF of Residuals
Results 2

Blue: Fitted Model
Red: Original data

$R^2 = 0.88$
The chosen seasonal ARIMA model is:

(1,1,1) \times (1,1,1)_{15}

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar(1)</th>
<th>ma(1)</th>
<th>sar(1)</th>
<th>sma(1)</th>
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<tbody>
<tr>
<td></td>
<td>0.5554</td>
<td>-0.8761</td>
<td>0.2256</td>
<td>-0.802</td>
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<td>se</td>
<td>0.0508</td>
<td>0.0344</td>
<td>0.0358</td>
<td>0.023</td>
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</table>
Modeling All Frames: Log Seasonal Model

Blue: Fitted Model
Red: Original data

$R^2 = 0.99$
Another approach to consider while modeling video traffic is to separate the three video frame types as individual models:

- Create separate models for I-Frames, P-Frames and B-Frames
- Compare the final results with the one obtained from the previous approach
Modeling I Frames 1

I-Frames

0 20 40 60 80 100 120 140 160
0 500 1000 1500 2000 2500 3000 3500

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Modeling I Frames 2

Series IFrames

Only one correlation at lag = 1

ignore
Modeling I Frames 3

Series IFrames

Only one correlation at lag = 1

ignore
The Proposed models are: ARIMA(0,0,1), ARIMA(1,0,0), ARIMA(1,0,1)

Determine the best model based on **AIC (Akaike's Information Criterion)**

\[
AIC = 2k – 2 \ln L \\
= 2k + n[\ln(2\pi \text{ RSS}/n) + 1]
\]

Here \( k \) is the number of parameters and \( L \) is the “goodness of the model”

**RSS** = Residual sum of squares

Lower AIC \( \Rightarrow \) Smaller number of parameters and lower errors

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[1]: http://en.wikipedia.org/wiki/Akaike_information_criterion
Proposed models: ARIMA(0,0,1), ARIMA(1,0,0), ARIMA(1,0,1)

Now we determine the best model based on AIC index (*lower is better*)

- ARIMA(0,0,1) [AIC] = 1922.87
- ARIMA(1,0,0) [AIC] = 1913.42
- ARIMA(1,0,1) [AIC] = 1913.82
Modeling I Frames 5

Standardized Residuals

ACF of Residuals
Modeling I Frames 6

Coefficients:

<table>
<thead>
<tr>
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<th>ar (1)</th>
<th>intercepts</th>
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<tr>
<td>se</td>
<td>0.0733</td>
<td>54.839</td>
</tr>
</tbody>
</table>
Modeling I Frames: Results

Blue: Fitted Model
Red: Original data

$R^2 = 0.98$
Modeling P Frames 1
Modeling P Frames 2

Series PFrames

Trails off at lag = 1

ignore
Modeling P Frames 3

Series PFrames

- Trails off at lag = 1
- Cuts off at lag = 2
- Ignore

Washington University in St. Louis  WiMAX AATG Meeting, Hawaii, Jan 31, 2008  ©2008 Raj Jain
ACF: Trails off at lag = 1
PACF: Trails off at lag = 1, cuts off at lag = 2
ARIMA models
- ARIMA(1,0,1) \( \Rightarrow \) AIC = 7724.41 √
- ARIMA(2,0,0) \( \Rightarrow \) AIC = 7733.21

Model coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar (1)</th>
<th>ma(1)</th>
<th>intercepts</th>
</tr>
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<tbody>
<tr>
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</table>
Modeling P Frames 5

Standardized Residuals

ACF of Residuals
Modeling P Frames 6

Blue: Fitted Model
Red: Original data

$R^2 = 0.81$
Modeling B Frames 1

B-Frames

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Modeling B Frames 2

Series BFrames

ACF

Lag

ignore
Modeling B Frames 2

Series B Frames

Trails off at lag = 1
Modeling B Frames 3

Series BFrames

- Trails off at lag = 1
- Cuts off or Trails off at lag = 5
- Ignore
ARIMA Models

- ARIMA(1,0,1) \(\Rightarrow\) AIC = 15315.06
- ARIMA(5,0,0) \(\Rightarrow\) AIC = 15187.51
- ARIMA(5,0,1) \(\Rightarrow\) AIC = 15186.66 \(\checkmark\)

<table>
<thead>
<tr>
<th></th>
<th>ar (1)</th>
<th>ar (2)</th>
<th>ar (3)</th>
<th>ar (4)</th>
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<th>intercepts</th>
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<td>13.3287</td>
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</tbody>
</table>
Modeling B Frames 5

Standardized Residuals

ACF of Residuals
Modeling B Frames 6

Blue : Fitted Model
Red : Original data

$R^2 = 0.89$
Combining I, P, B Models 1

- To get a full model for all frames, a combination of the model will be presented.

- Points to consider:
  - The three models represented for I-Frames, P-Frames and B-Frames did not show the seasonal trend
  - Models are easier to implement than the All-Frames models
Combining I, P, B Models 2

Blue : Fitted Model
Red  : Original data

$R^2 = 0.82$
In this presentation we showed that a good model for video traffic is achievable using ARIMA model.

Seasonal ARIMA is a better approach to get a better video traffic model.

Creating individual models for each frame type is not necessary a better approach.
Future Work

- Study and evaluate more video traces considering different encoding settings, GOP sizes, and video scene nature
- Present a traffic model that represents the variety of video streaming traffic
- Implement the model in NS2
- Present simulation results