Generalized Fairness

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Generalized Fairness

- Real-time applications need non-zero Minimum Cell Rate (MCR)
- In TM4, Distribution of excess bandwidth (fairness) is implementation specific.
- TM4.0 has five examples of fair distribution
- We have shown that two of the examples are not meaningful and have proposed a sixth example that is a superset of the remaining three definitions
- We developed a switch algorithm that implements the proposed definition
Pricing Function

- \( T = \) Small time interval, \( W = \) Number of bits
- \( R = \) Average rate \( W/T \)
- Cost \( C = f(W,R) \). If \( C \) is restricted to continuous differentiable functions of type: \( C = \sum_{ij} a_{ij} W^i R^j \)
- For all values of \( W \) and \( R \):
  - \( C \geq 0 \quad \partial C/\partial W \geq 0 \quad \partial C/\partial R \geq 0 \)
  - \( \partial (C/W)/\partial W \leq 0 \) [Economy of Scale]
  - \( \partial (C/R)/\partial R \leq 0 \) [Economy of Scale]
- The only function that satisfies all 5 conditions is:
  \[ C = a_{00} + a_{10} W + a_{01} R + a_{11} WR \]
A Simple Pricing Fn

- \( f() \) is non-decreasing w.r.t to \( W \)
- \( f() \) is non-increasing w.r.t to \( T \) \( \Rightarrow \) non-decreasing w.r.t \( R \)

- A simple function satisfying these requirements is:
  \[ C = c + wW + rR \]
  Here, \( c = \) Fixed cost per connection
  \( w = \) Cost per bit (How much)
  \( r = \) Cost per Mbps (How fast)

- This cost function implies that the excess bandwidth
  should be allocated using the proposed generalized
  fairness function
1. $B(i) = B/n$

2. $B(i) = MCR(i) + (B-M)/n$

3. $B(i) = \text{Max}\{MCR(i), \text{Max-Min Share}\}$

4. $B(i) = B*(MCR(i)/M)$

5. $B(i) = w(i)\times B/\text{Sum}(w(j))$

- Definition 5 does not always guarantee MCR
- Definition 3 may result in total of fair share being more than the capacity

- Notation: $n = \# \text{ of active VCs bottlenecked here}$
  $B = \text{Bandwidth available for the bottlenecked VCs}$
  $M = \Sigma MCR(I)$
General Definition

- Fair Share
  \[ B(i) = MCR(i) + \frac{w(i) (B - M)}{\sum_{j=1,n} w(j)} \]

- This definition is a superset of 1, 2, 4 in TM4.0

- Always ensures MCR

- If all vendors implement the generalized fairness, the network manager can select network-wide their desired fairness criteria by appropriately setting weights.
Mapping to TM 4.0

- $w(i) = w, \ MCR(i)=0: \ B(i) = B/n$
  This is Definition 1 (Max-min Fair).

- $w(i) = w: \ B(i) = MCR(i) + (B - M)/n$
  This is Definition 2 (MCR plus equal share)

- $w(i) = MCR(i)$:
  $B(i) = MCR(i) + (B-M) \frac{MCR(i)}{M}$
  $\quad = B^* \frac{MCR(i)}{M}$
  This is Definition 4 (Proportional to MCR)