

Generalized Fairness

Raj Jain

The Ohio State University

Columbus, OH 43210

Jain@CIS.Ohio-State.Edu

<http://www.cis.ohio-state.edu/~jain/>

Generalized Fairness

- ❑ Real-time applications need non-zero Minimum Cell Rate (MCR)
- ❑ In TM4, Distribution of excess bandwidth (fairness) is implementation specific.
- ❑ TM4.0 has five examples of fair distribution
- ❑ We have shown that two of the examples are not meaningful and have proposed a sixth example that is a superset of the remaining three definitions
- ❑ We developed a switch algorithm that implements the proposed definition

Pricing Function

- T = Small time interval, W = Number of bits
 R = Average rate W/T
- Cost $C = f(W, R)$. If C is restricted to continuous differentiable functions of type: $C = \sum_{ij} a_{ij} W^i R^j$
- For all values of W and R :
 - $C \geq 0$ $\partial C / \partial W \geq 0$ $\partial C / \partial R \geq 0$
 - $\partial(C/W) / \partial W \leq 0$ [Economy of Scale]
 - $\partial(C/R) / \partial R \leq 0$ [Economy of Scale]
- The only function that satisfies all 5 conditions is:

$$C = a_{00} + a_{10}W + a_{01}R + a_{11}WR$$

A Simple Pricing Fn

- $f()$ is non-decreasing w.r.t to W
 $f()$ is non-increasing w.r.t to $T \Rightarrow$ non-decreasing w R
- A simple function satisfying these requirements is:
$$C = c + w W + r R$$

Here, c = Fixed cost per connection
 w = Cost per bit (How much)
 r = Cost per Mbps (How fast)
- This cost function implies that the excess bandwidth should be allocated using the proposed generalized fairness function

TM4.0 Definitions

1. $B(i) = B/n$

2. $B(i) = \text{MCR}(i) + (B-M)/n$

3. $B(i) = \text{Max}\{\text{MCR}(i), \text{Max-Min Share}\}$

4. $B(i) = B * (\text{MCR}(i)/M)$

5. $B(i) = w(i) * B / \text{Sum}(w(j))$

□ Definition 5 does not always guarantee MCR

□ Definition 3 may result in total of fair share being more than the capacity

□ Notation: $n = \#$ of active VCs bottlenecked here

$B =$ Bandwidth available for the bottlenecked VCs

$$M = \sum \text{MCR}(I)$$

General Definition

- Fair Share

$$B(i) = MCR(i) + \frac{w(i) (B - M)}{\sum_{j=1,n} w(j)}$$

- This definition is a superset of 1, 2, 4 in TM4.0
- Always ensures MCR
- If all vendors implement the generalized fairness, the network manager can select network-wide their desired fairness criteria by appropriately setting weights.

Mapping to TM 4.0

- $w(i) = w, \text{MCR}(i)=0: B(i) = B/n$
This is Definition 1 (Max-min Fair).
- $w(i) = w: B(i) = \text{MCR}(i) + (B - M)/n$
This is Definition 2 (MCR plus equal share)
- $w(i) = \text{MCR}(i):$
$$B(i) = \text{MCR}(i) + (B-M) \text{MCR}(i) / M$$
$$= B^* (\text{MCR}(i)/M)$$

This is Definition 4 (Proportional to MCR)