Chapter 4
Digital Data Communication Techniques

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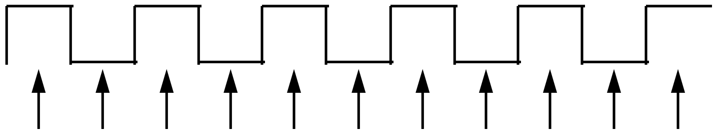
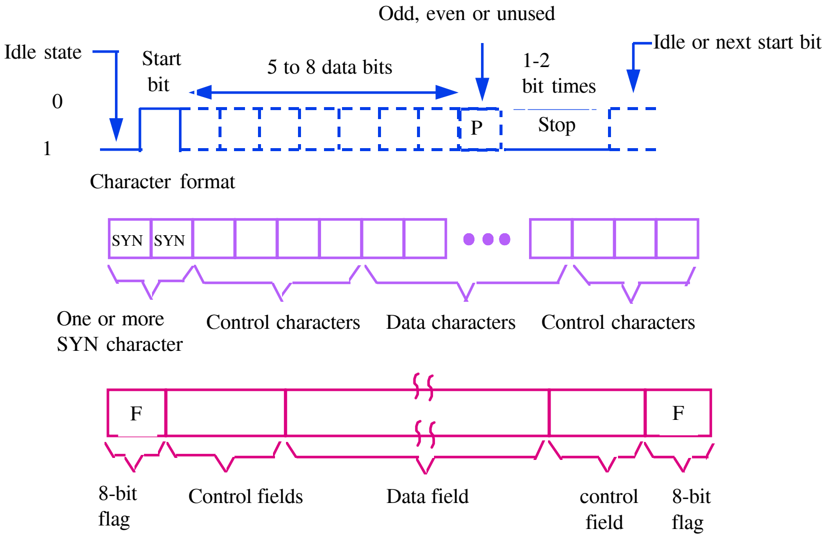
- Asynchronous vs Synchronous Transmissions
- Error Detection: Parity, LRC, CRC

Clock Synchronization

- Suppose, data rate = 10 kbps
  One bit = 0.1 ms
- Clock rate is 1% faster,
  Sampling every 0.099 ms
- After 50 bits: 50% away from center ⇒ Error



Asynchronous vs Synchronous



Asynchronous Transmission

- Used for short bit sequences
- Idle = No signal, negative voltage, 1
- One Start bit, 7 or 8 data bits
- One parity bit: Odd, Even, None
- Minimum Gap = Stop bits = 1, 1.5, or 2 bits
- Efficiency = data bits/total bits
  8N1 = 1 Start bit + 8 Data bits + 1 Stop bit + 1 parity bit (even though the parity is not being used by this site)
  ⇒ 8/(1+8+1+1) = 73%
- Faster clock: 7% ⇒ 56% off on 8th bit ⇒ Error
- Framing error ⇒ False start/end of a frame

Synchronous Transmission

- Used for longer bit sequences
- Requires clock transmission
  Use codes with clock information (Manchester)
- Begining of block indicated by a preamble bit pattern called "Syn"
- End of block indicated by postamble bit pattern
- Character-oriented transmission: Data in 8-bit units
- Bit-oriented transmission: Preamble = Flag
- Efficiency: Data bits/(Preamble+Data+Postamble)
- High-Level Data Link Control (HDLC) uses bit-oriented synchronous transmission. 8 bit of overhead for 1000 data bits
  ⇒ 48/1048 = 4.6% overhead

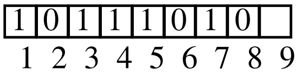
Probability of Frame Error

- Probability of bit error = p
- Number of bits per frame = n
- Probability of no error in a bit = $1-p$
- Probability of i errors in a frame = ${}^{n}C_i\, p^i\, (1-p)^{n-i}$
  ${}^{n}C_i = n!/\{i!\,(n-i)!\}$
- Probability of no error in a frame = $(1-p)^n$
- Probability of error in a frame = $1-(1-p)^n$

Error Detection Techniques

- Parity Checks
- Longitudinal redundancy checks
- Cyclic redundancy checks

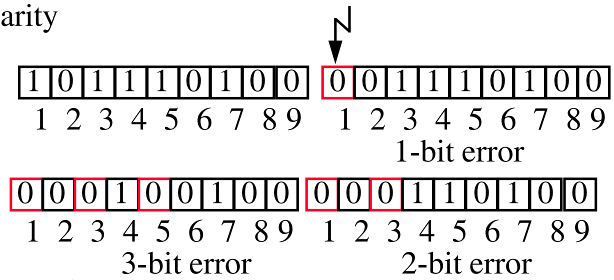
Parity Checks



1 0 1 1 1 0 1 0

1 2 3 4 5 6 7 8 9

- ❑ Odd Parity



1 0 1 1 1 0 1 0 0 | 0 0 1 1 1 0 1 0 0

1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9

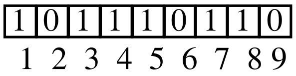
1-bit error

0 0 0 1 0 0 1 0 0 | 0 0 0 1 1 0 1 0 0

1 2 3 4 5 6 7 8 9 | 1 2 3 4 5 6 7 8 9

3-bit error | 2-bit error

- ❑ Even Parity

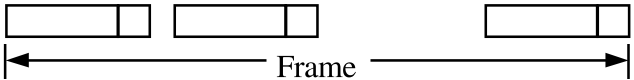


1 0 1 1 1 0 1 1 0

1 2 3 4 5 6 7 8 9

Probability of Error with Parity

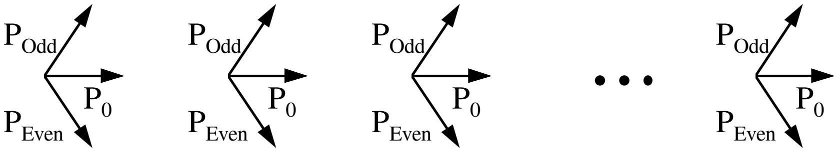
- ❑ Let, n= Number of bits per character (= N_B in the book)
  n includes the parity bit
  N=number of characters per frame (=N_C in the book)
- ❑ Probability of no errors/Char $P_0 = (1-P_B)^n$
  P_B = bit error probability = p
- ❑ Probability of i bit errors/Char = ${}^nC_i \, P_B^i (1-P_B)^{n-i}$
- ❑ Probability of odd # of bits errors/Char
  $P_{Odd} = n\, PB(1-P_B)^{n-1} + {}^nC_3\, P_B^3(1-P_B)^{n-3} + {}^nC_5\, P_B^5(1-P_B)^{n-5} +$
- ❑ Probability of undetected (even # of bits) errors/Char
  $P_{Even} = {}^nC_2\, P_B^2(1-P_B)^{n-2} + {}^nC_4\, P_B^4(1-P_B)^{n-4} + {}^nC_6\, P_B^6(1-P_B)^{n-6} +$



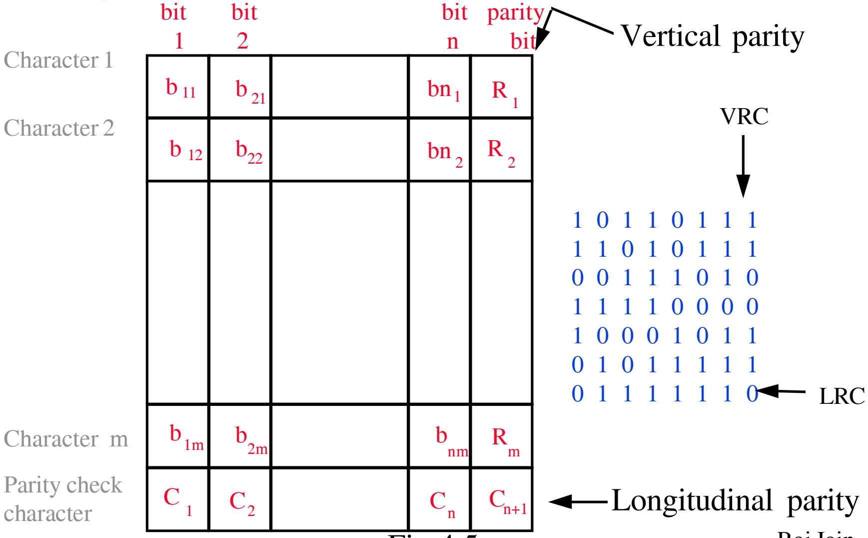
Frame

Probabilities (Cont)

- Probability of no errors/frame = $P_1 = (1-P_B)^{nN}$
- P(Undetected Error) = P_{UE}
  = P(k even errors and N-k no errors)
  = $\Sigma^N_{k=1} {}^N C_k \, P_{Even}{}^k P_0{}^{(N-k)}$
- Probability of detected errors P_{DE}
  = *1-P(no errors) - P(undetected errors)*
  = $1- P_1 - P_{UE}$



Longitudinal Redundancy Checks (LRC)



	bit 1	bit 2		bit n	parity bit
Character 1	b_{11}	b_{21}		bn_1	R_1
Character 2	b_{12}	b_{22}		bn_2	R_2
Character m	b_{1m}	b_{2m}		b_{nm}	R_m
Parity check character	C_1	C_2		C_n	C_{n+1}

1	0	1	1	0	1	1	1
1	1	0	1	0	1	1	1
0	0	1	1	1	0	1	0
1	1	1	1	0	0	0	0
1	0	0	0	1	0	1	1
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0

Fig 4.5

Check Digit Method

- ❑ Make number divisible by 9

Example: 823 is to be sent

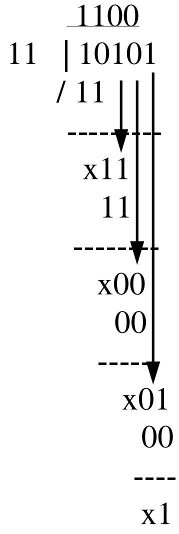
1. Left-shift: 8230
2. Divide by 9, find remainder: 4
3. Subtract remainder from 9: 9-4=5
4. Add the result of step 5 to step 1: 8235
5. Check that the result is divisible by 9.

Detects all single-digit errors: 7235, 8335, 8255, 8237

Detects several multiple-digit errors: 8765, 7346

Does not detect some errors: 7335, 8775, ...

Modulo 2 Arithmetic



```
  1111     11001            1100
+1010    ×    11      11  | 10101
--------   --------          / 11
  0101      11001            -------
           11001              x11
           ---------            11
           101011            ------
                                x00
                                 00
                              -----       010     2
                                  x01     011     3
                                   00     ----    --
                                 ----     001     1  Mod 2
                                   x1     101     5  Binary
```

Cyclic Redundancy Check (CRC)

- ❑ **Binary Check Digit Method**
- ❑ Make number divisible by P=110101 (n+1=6 bits)

Example: M=1010001101 is to be sent

1. Left-shift M by n bits 2^nM= 101000110100000

2. Divide 2^nM by P, find remainder: R=01110

~~3. Subtract remainder from P~~

4. Add the result of step 2 to step 1 : T=101000110101110

5. Check that the result T is divisible by P.

Detects all single-bit errors

Detects several multiple-bit errors

Does not detect some errors

Modulo 2 Division

```
       Q=1101010110
P=110101)101000110100000=2ⁿM
         110101
          111011
          110101
           011101
           000000
            111010
            110101
             011111
             000000
              111110
              110101

                                010110
                                000000
                                 101100
                                 110101
                                  110010
                                  110101
                                   001110
                                   000000
                                    01110 = R
```

Checking At The Receiver

```
      1101010110
110101)101000110101110
      110101
       111011                 010111
       110101                 000000
        011101                 101111
        000000                 110101
         111010                 110101
         110101                 110101
          011111                 00000
          000000
           111110
           110101
```

Polynomial Representation

- Number the bits 0, 1, ..., from right

  $b_n b_{n-1} b_{n-2} \ldots b_3 b_2 b_1 b_0$

  $b_n x^n + b_{n-1} x^{n-1} + b_{n-2} x^{n-2} + \ldots + b_3 x^3 + b_2 x^2 + b_1 x + b_0$

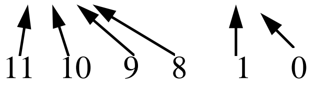
- Example:

  543210

  ↓↓↓↓↓

  110101 = $x^5 + x^4 + x^2 + 1$

  1101 1001 0011 = $x^{11} + x^{10} + x^8 + x^7 + x^4 + x + 1$



Cyclic Redundancy Check (CRC)

Polynomial Division Method

Make $T(x)$ divisible by $P(x) = x^5+x^4+x^2+1$ (Note: n=5)

Example: M=1010001101 is to be sent

$M(x) = x^9+x^7+x^3+x^2+1$

1. Multiply M(x) by x^n, $x^nM(x)= x^{14}+x^{12}+x^8+x^7+x^5+$ ….

2. Divide $x^nM(x)$ by $P(x)$, find remainder:
   $R(x)$=01110=x^3+x^2+x

4. Add the remainder $R(x)$ to $x^nM(x)$:
   $T(x)= x^{14}+x^{12}+x^8+x^7+x^5+x^3+x^2+x$

5. Check that the result $T(x)$ is divisible by $P(x)$.

Transmit the bit pattern corresponding to $T(x)$:

101000110101110

Popular CRC Polynomials

- CRC-12: $x^{12}+x^{11}+x^3+x^2+x+1$
- CRC-16: $x^{16}+x^{15}+x^2+1$
- CRC-CCITT: $x^{16}+x^{12}+x^5+1$
- CRC-32: Ethernet, FDDI, ...
  $x^{32}+x^{26}+x^{23}+x^{22}+x^{16}+x^{12}+x^{11}$
  $+x^{10}+x^8+x^7+x^5+x^4+x^2+x+1$

Even number of terms in the polynomial

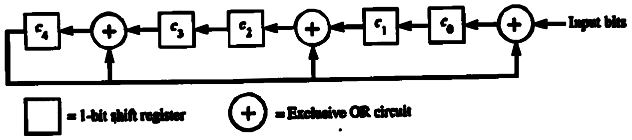
⇒ Polynomial is divisible by 1+x

⇒ Will detect all odd number of bit errors

Errors Detected by CRC

- ☐ All single bit errors
- ☐ Any burst error of length n bits or less, n=degree of the polynomial
- ☐ Most larger burst errors
  P(undetected burst errors|error has ocurred) = 2^{-n}
- ☐ Any odd number of errors if P(x) has 1+x as a factor, i.e., has even number of terms
- ☐ Any double bit errors as long as P(x) has a factor with 3 terms, e.g., $(1+x^4+x^9)(....)$

Shift-Register Implementation



(a) Shift-register implementation

$x^5+x^4+x^2+1$

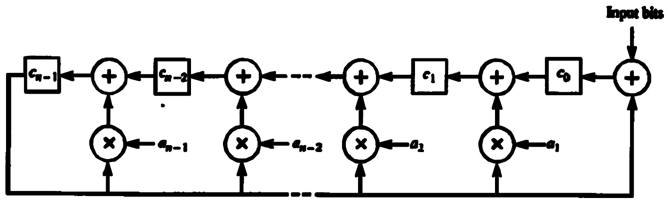
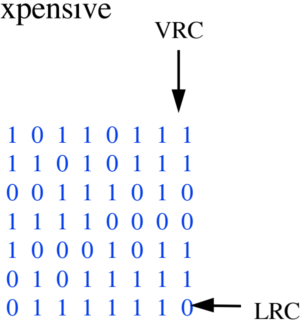


FIGURE 4.7. General CRC architecture to implement divisor $1 + a_1X + a_2X^2 + \cdots + a_{n-1}X^{n-1} + X^n$.

Fig 4.7, 4.6a

Error Correcting Codes (ECC)

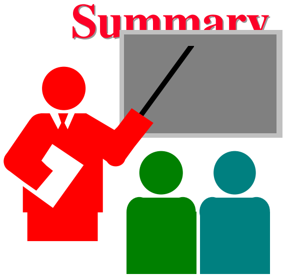
- ❑ Example:
  VRC+LRC will correct all single bit errors
- ❑ Forward error correction (FEC)
  Used if retransmission expensive



1	0	1	1	0	1	1	1
1	1	0	1	0	1	1	1
0	0	1	1	1	0	1	0
1	1	1	1	0	0	0	0
1	0	0	0	1	0	1	1
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0

Fig 4.5b

Summary



- ❑ Asynchronous and Synchronous transmission
- ❑ Parity, LRC, VRC, CRC
- ❑ CRC Polynomials

Homework

- Exercise: 4.1
  In b assume flags are included in 48 control bits.
  In d, assume syn is included in 9 control characters.
  Assume 7 data bits + 1 parity.
- Exercise: 4.13ab
  P(Undetected error) = 1- {P(No error) + P(Detected Errors)}
- Exercises: 4.14, 4.16

- Due next class