EE273 Lecture 2
Wires

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Announcements

- Please check course webpage regularly for announcements
  - http://eeclass.stanford.edu/ee273
- Please register for class

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Today's Assignment

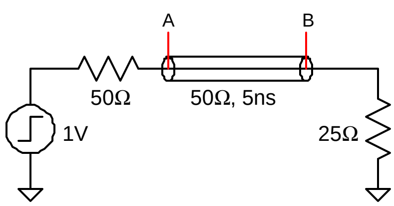
- Reading
  - Sections 3.4, 3.6, and 3.7
  - Complete before class on Wednesday 1/16

Outline

- Electrical properties of wires
- Equivalent model of an ideal transmission line
- Derivation of the Telegrapher's equation
- Lossy lines

Last Time

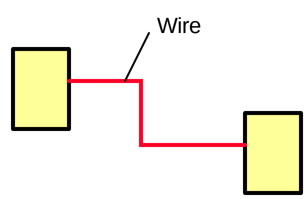
- Three rules of transmission lines
  - Waves propagate down the line (in both directions)
  - Waves reflect unless terminated
  - The voltage on the line is the *superposition* of these waves



$$k_r = \frac{Z_T - Z_0}{Z_T + Z_0} =$$

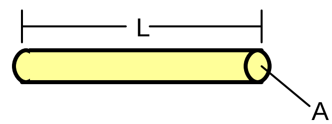
Wires in Digital Systems

- Physically wires are
  - Microstrip and stripline on printed-circuit cards and backplanes
  - Conductors in cables, cable assemblies
  - Connectors
- We tend to treat them as ***ideal wires***
  - no delay (equipotential)
  - no capacitance, inductance, or resistance
- They are ***not*** ideal
- To build reliable systems we need to understand their properties and behavior



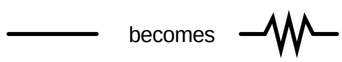
Resistance of Wires

- Most *real* wires have resistance
- Depends on
  - material (resistivity)
  - length
  - cross section
- Causes
  - delay
  - loss



$$R = \frac{rL}{A}$$

Material	ρ (nΩ-m)
Ag	16
Cu	17
Au	22
Al	27

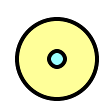


Capacitance of Wires

- Real wires have capacitance
  - line charge
  - parallel plate
  - fringing
- To compute
  - assume Q
  - compute E field
  - integrate to get V
- Think of the energy stored in the E field

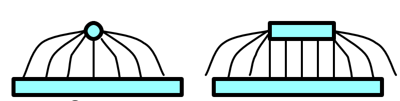
$$C = \frac{Q}{V}$$

$$E = \frac{Q}{2pr}$$



$$C = \frac{2pe}{\log\left(\frac{r_o}{r_i}\right)}$$

$$C = \frac{2pe}{\log\left(\frac{s}{r}\right)}$$



$$\frac{2pe}{\log\left(2s/r\right)}$$

$$C = \frac{we}{d} + \frac{2pe}{\log\left(2s/r\right)}$$

log is natural logarithm, base e

Inductance of Wires

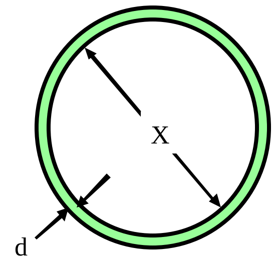
- Real wires have inductance

$$L = \frac{f}{I}$$

- In a homogenous medium

$$CL = em$$

- Inductance is a purely geometric property of a ***closed*** circuit
- Think of the energy stored in the magnetic field



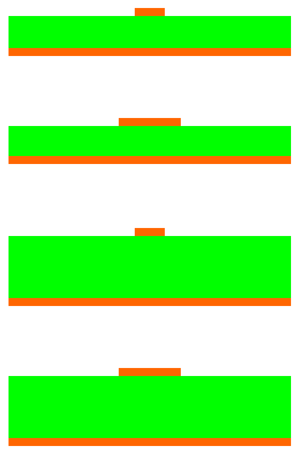
$$L = 3.96 \cdot 10^{-10}\, X \left(\ln\left[\frac{8X}{d}\right] - 2 \right)$$

Some Example Wires

Type	W	R	C	L
On chip	0.6μm	150kΩ/m	200pf/m	600nH/m
PC Board	150μm	5Ω/m	100pf/m	300nH/m
24AWG pair	511μm	0.08Ω/m	40pf/m	400nH/m

Scale model of a line has different R, but same L and C per unit length

Qualitative L and C



$L = L_1$, $C = C_1$, $R = R_1$

L = , C = , R =

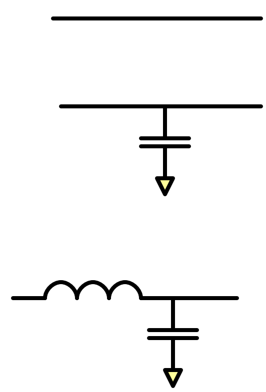
L = , C = , R =

L = , C = , R =

Wire Models

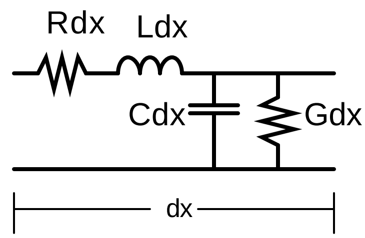
- In a particular situation, we create a *model* of a wire that captures the properties we need
  - ideal
  - lumped L, R, or C
  - RC transmission line
  - LC transmission line
  - General LRCG transmission line
- Model to use depends on frequency

$$f_0 = \frac{R}{2pL}$$

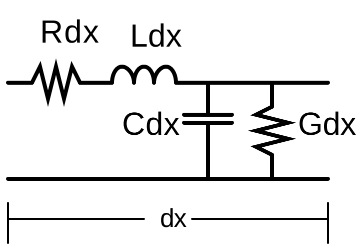


LRCG Wire Model

- Model an *infinitesimal* length of wire, dx, with lumped components
  - L, R, C, and G



Transmission Line Equations



Drop across R and L $$\frac{\partial V}{\partial x} = RI + L\frac{\partial I}{\partial t}$$

Current into C and G $$\frac{\partial I}{\partial x} = GV + C\frac{\partial V}{\partial t}$$

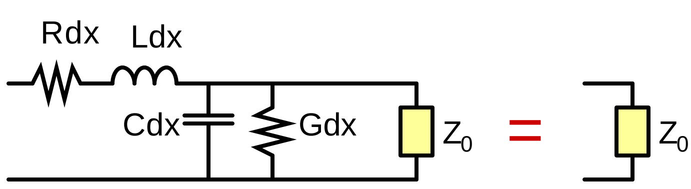
$$\frac{\partial^2 V}{\partial x^2} = RGV + (RC + LG)\frac{\partial V}{\partial t} + LC\frac{\partial^2 V}{\partial t^2}$$

Solving this equation gives us two key properties of the line

- Impedance
  - I-V relationship at terminal
- Propagation constant
  - How a signal propagates down the line
    - How fast
    - How much distortion

Impedance

- An infinite length of LRCG transmission line has an *impedance* Z_0
- Driving a line *terminated* into Z_0 is the same as driving Z_0
- In general Z_0 is complex and frequency dependent
- For LC lines its real and independent of frequency



$$Z_0 = \left(\frac{R + Ls}{G + Cs} \right)^{\frac{1}{2}}$$

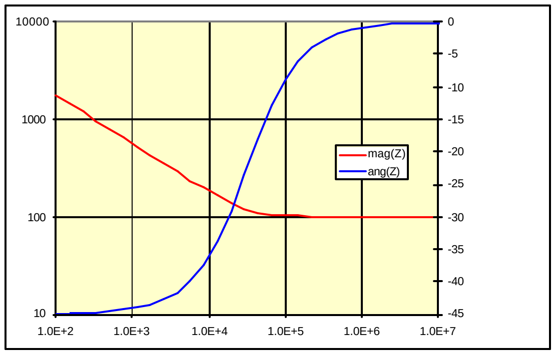
$$Z_0 = \left(\frac{L}{C} \right)^{\frac{1}{2}}$$

At high frequency (LC lines)

Example, 24AWG Pair

- f_0 = 33kHz
- Below f_0, line is RC
- Above f_0, line is LC

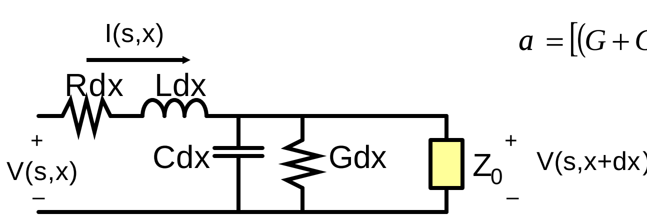
$$Z_0 = \left(\frac{.08 + 400 \times 10^{-9} \times 2pfj}{40 \times 10^{-9} \times 2pfj} \right)^{\frac{1}{2}}$$



Propagation Constant

- Using impedance, we can solve for V(s,x)
- Propagation is governed by a constant, α
  - real part is attenuation
  - imaginary part is phase shift
    - velocity^{-1}
    - v = $(LC)^{-1/2}$

$$\frac{\partial V(s)}{\partial x} = -(R + Ls)I(s)$$
$$= -(R + Ls)V(s)/Z_0$$
$$= -[(G + Cs)(R + Ls)]^{\frac{1}{2}} V(s)$$
$$V(s,x) = V(s,0)\exp(-ax)$$
$$= V(s,0)\exp(-a_{RE}x)\exp(-a_{IM}x)$$
$$a = [(G + Cs)(R + Ls)]^{\frac{1}{2}}$$



Lossless LC Lines

- If R and G are negligible
  - line is lossless (no dissipation)
  - governed by the *wave equation*
- Waves propagate down the line in both directions without distortion
- Line is described by its ***impedance*** and ***velocity***
- For a general LCRG line we have ***impedance*** and ***propagation constant***

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

$$V_f(x,t) = V\left(0, t - \frac{x}{v}\right)$$

$$V_r(x,t) = V\left(x_{\max}, t - \frac{x_{\max} - x}{v}\right)$$

$$v = (LC)^{-\frac{1}{2}}$$

$$Z_0 = \left(\frac{L}{C}\right)^{\frac{1}{2}}$$

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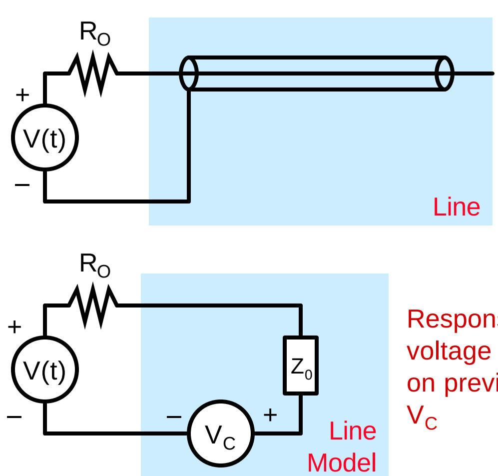
Driving a Line - Equivalent Circuit



Response of line to voltage source depends on previous state of line, V_C

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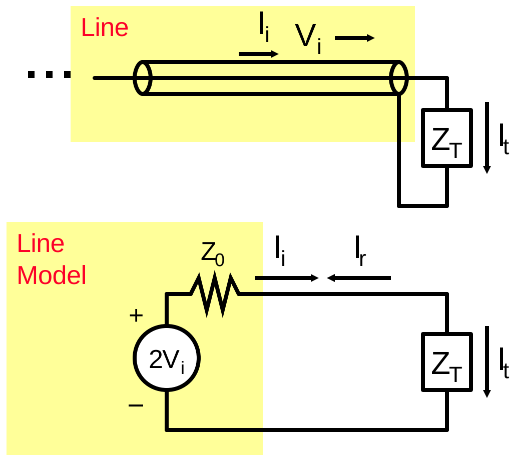
Termination - Equivalent Circuit



Reflections and The Telegrapher's Equation

- Incident wave determines V_i, I_i
- Use equivalent circuit to solve for V_T, I_T
- Use superposition to calculate V_r, I_r

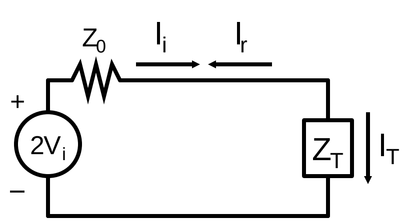
$$I_T = \frac{2V_i}{Z_0 + Z_T}$$

$$I_r = I_i - I_T$$

$$I_r = \frac{V_i}{Z_0} - \frac{2V_i}{Z_0 + Z_T}$$

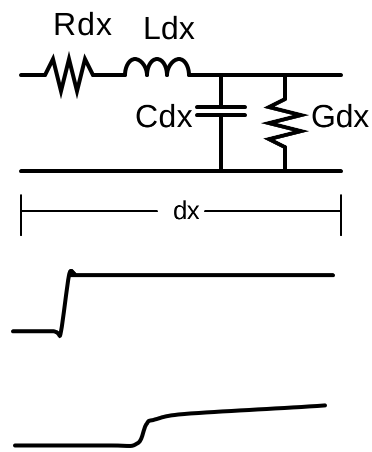
$$I_r = \frac{V_i}{Z_0}\left(\frac{Z_T - Z_0}{Z_T + Z_0}\right)$$

$$\frac{I_r}{I_i} = \frac{V_r}{V_i} = \left(\frac{Z_T - Z_0}{Z_T + Z_0}\right)$$



Lossy Transmission Lines

- LC lines with resistance and conductance
  - propagation mostly by wave
  - some by diffusion
- R and G dissipation
  - reduces the amplitude of the signal
  - disperses the signal
    - fast rise to AC attenuation
    - slow tail to DC attenuation
- Resistance and conductance depend on frequency
  - we will ignore this for now



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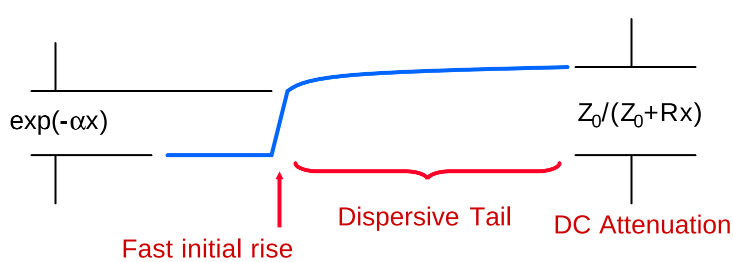
23

Zero-th Order Waveform



$$a = \left[(G+Cs)(R+Ls)\right]^{\frac{1}{2}}$$

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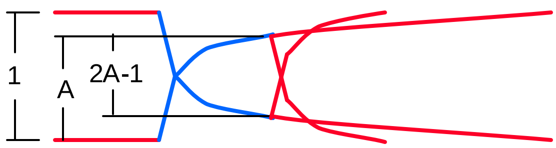
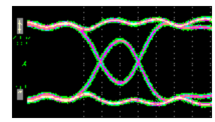
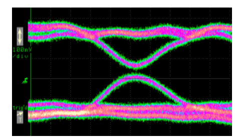
24

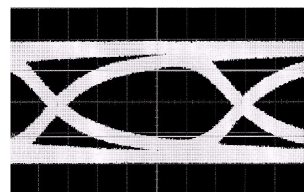
Q: So why worry about attenuation?
A: It closes the eye opeing!

- Critical parameter is what fraction of swing, A is achieved in one bit time
- Eye opening is reduced to B = 2A-1
- No eye opening at 50% attenuation
- Significant degradation of margins at lower levels of attenuation





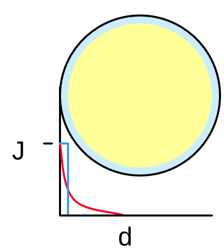




Skin Effect Resistance

- Beauty is only skin deep - so is current
  - current density drops off exponentially with depth
- Skin depth decreases with frequency, $f^{1/2}$
- Model as if all current flowed in δ-thick outer layer of conductor

$$d = (pfms)^{-1/2}$$



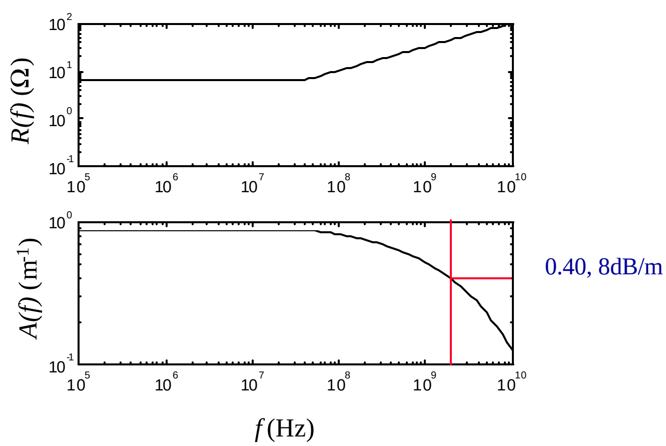
$$J = \exp\left(-\frac{d}{d}\right)$$

Skin-Effect Resistance

- Effect does not occur until frequency, f_s, at which skin depth equals conductor radius
- Above f_s, R and A increase as the square-root of frequency

$$R(f) = \frac{R_{DC}}{2}\left(\frac{f}{f_s}\right)$$

Resistance and Attenuation of 5mil 0.5oz 50Ω Stripguide



Dielectric Absorption

- High frequency signals *jiggle* molecules in the insulator
  - insulator *absorbs* signal energy
- This effect is approximately linear with frequency and is modeled as a conductance
- Dielectric loss is often specified in terms of a *loss tangent,* tan(δ)

$$\tan\delta = \frac{G}{\omega C}$$

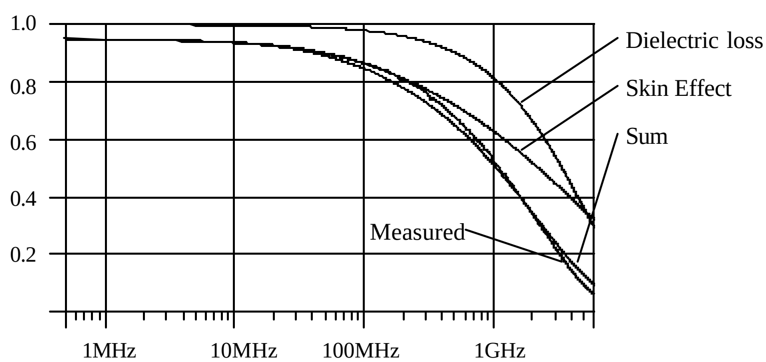
$$\alpha_D = \frac{GZ_0}{2}$$

$$= \pi f \tan\delta \sqrt{LC}$$

$$= \frac{\pi \sqrt{\varepsilon_r} f \tan\delta}{c}$$

material	tan δ
FR4	0.035
Polyimide	0.025
GETEK	0.010
Teflon	0.001

Skin effect resistance and dielectric absorption



1m 8mil 50Ω stripguide with GETEK dielectric

The Bd^2 Constant

- Suppose you can tolerate a certain attenuation, A_1
  - eye opening is 2A-1
- At a certain bandwidth, B_1, attenuation A is achieved with a distance of 1m
- As bandwidth is increased, resistance, and hence attenuation, increases as $B^{1/2}$
- So distance must be decreased by a proportional amount

$$A(B_1) = A_1$$

$$A(B,d) = A_1 d \left(\frac{B}{B_1} \right)^{1/2}$$

$$Bd^2 = B_1$$

Doubling distance cuts bandwidth by a factor of 4

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Next Time

- Transmission line wrapup
- Differential lines
- Multi-drop buses

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