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# CSE464

## Bergeron method

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# Bergeron Method (Graphical)

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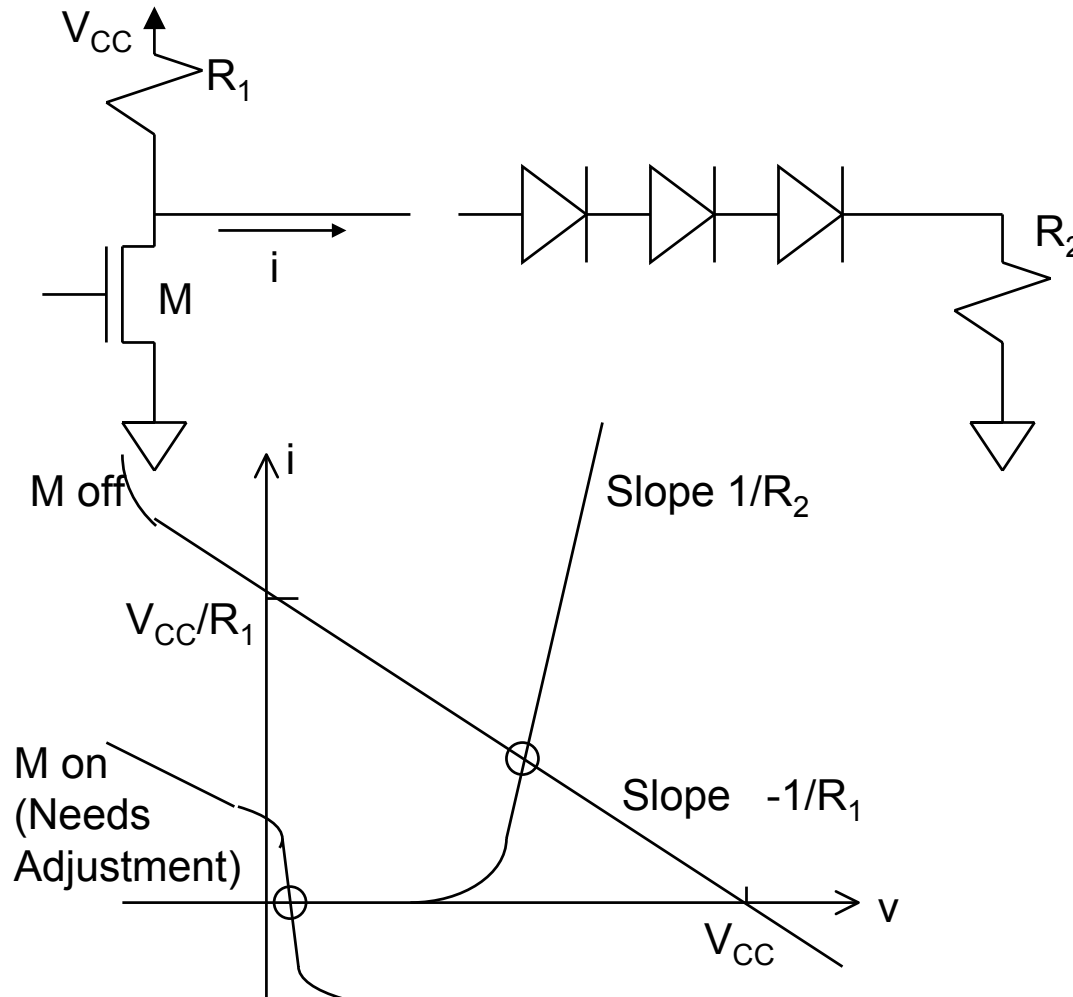
- Originally applied to study water hammer in hydraulic systems.
- Non-linear systems are difficult to handle analytically.
- Can be considered graphical solution for reflection coefficient, or application of load line and t-line output characteristic.
- *You must be able to determine circuit characteristics (V-I plots).*
- Be careful with step function (easier to analyze) vs non-zero rise time (realistic). Difference is small with linear system, but can be very important for non-linear system.

# Graphical Procedure in General

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- Plot output characteristic of driver
- Plot input characteristic of receiver
- Find intersection
  - Each plot represents all allowed or possible V-I combinations
  - The intersection is the only possible V-I combination for interconnection of driver and receiver
  - For positive resistance devices there must be exactly one intersection

# Graphical Solution: Simple Example



# Hints

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- Elements in series: add voltages
- Elements in parallel: add currents
- If slope of output characteristic is positive check your work (this would be negative resistance)
- If slope of input characteristic is negative check your work (this would be negative resistance)
- Pick scale so  $1/Z_o$  has slope 1,  $-1/Z_o$  has slope -1.
  - Easier plotting since lines are at 45 degrees
  - More accurate plots, not compressed along one axis

# Equations Justifying Bergeron's Method

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$$V(x, t) + Z_o \cdot I(x, t) = V_f\left(t - \frac{x}{v}\right) + V_r\left(t + \frac{x}{v}\right) + \frac{Z_o}{Z_o} \left( V_f\left(t - \frac{x}{v}\right) - V_r\left(t + \frac{x}{v}\right) \right)$$

$$V(x, t) + Z_o \cdot I(x, t) = 2 \cdot V_f\left(t - \frac{x}{v}\right)$$

Constant traveling from left to right

$$V(x, t) - Z_o \cdot I(x, t) = 2 \cdot V_r\left(t + \frac{x}{v}\right)$$

Constant traveling from right to left

If we know  $V(x, t) + Z_o \cdot I(x, t)$

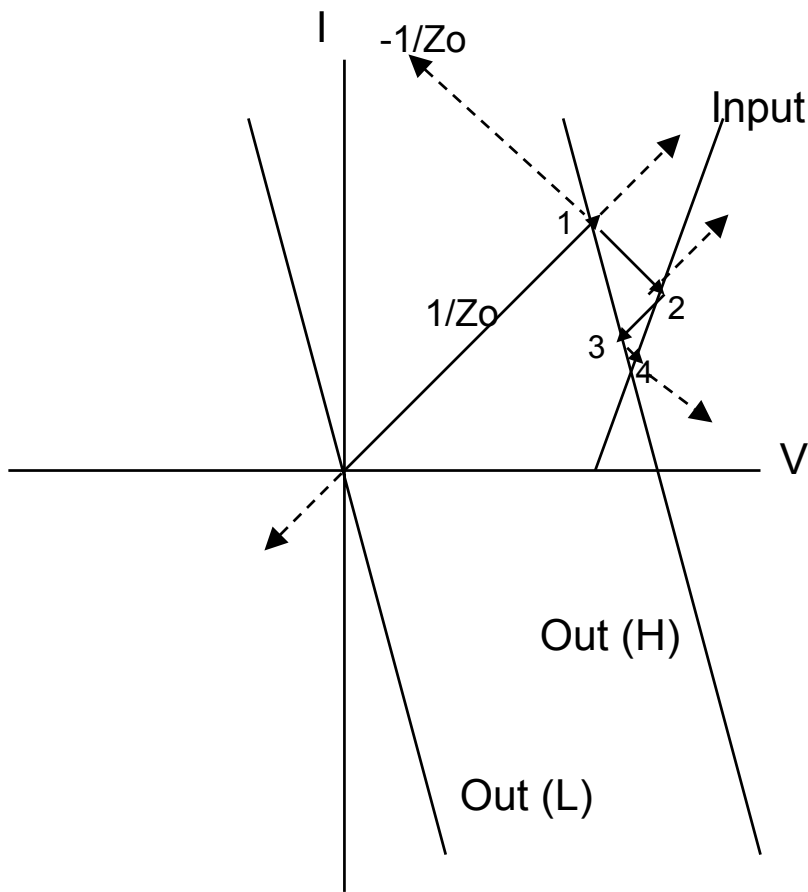
At  $x=0$ , then we know its value at  $x=l$ ,  
a time  $l/v$  later

Similarly in reverse direction

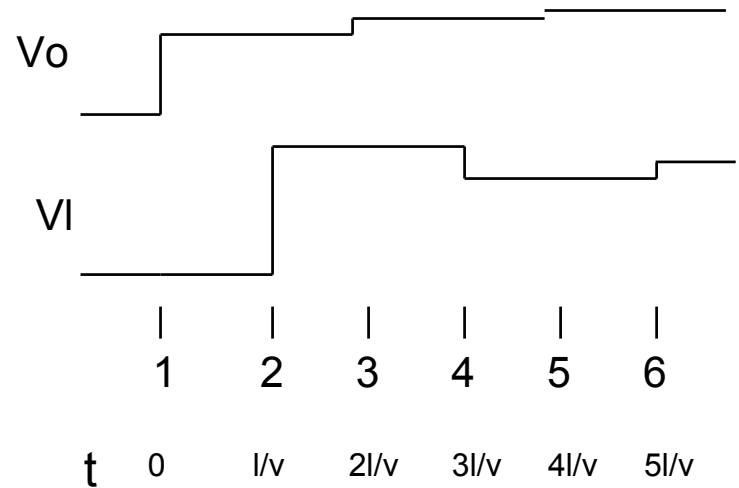
# Simple Example

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- $V_s=0V$  to  $1V$  step
- $R_s=20$  Ohms
- $Z_o=50$  Ohms
- Receiver is diode with forward voltage of  $0.8V$ , series resistance of  $20$  Ohms.

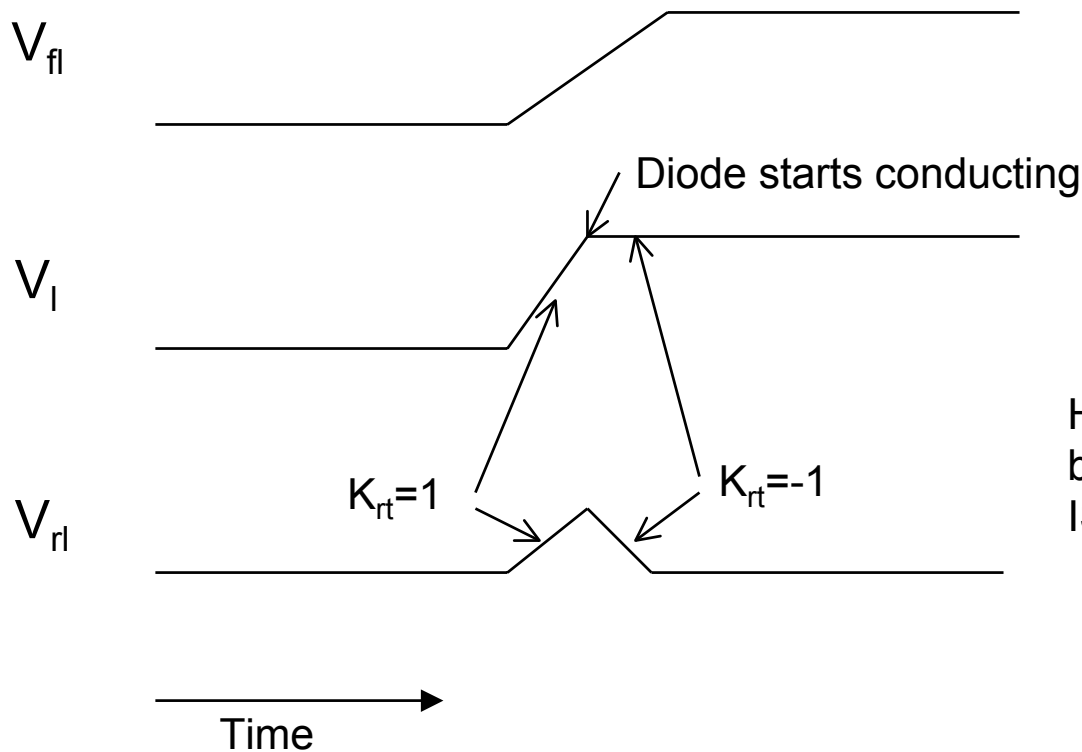


1.  $V_o(0^+)$
2.  $V_I(1/v^+)$
3.  $V_o(2/v^+)$
4.  $V_I(3/v^+)$



# Finite Rise-Time Signal and Diode-Clamp Termination

Ideal diode clamp, infinite resistance until conducting, then zero resistance



Half amplitude wave reflected back to source, can give severe ISI