CSE 559A: Computer Vision

Fall 2017: T-R: 11:30-1pm @ Lopata 101

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http://www.cse.wustl.edu/~ayan/courses/cse559a/

Sep 21, 2017
Recitation tomorrow (9/22) 10am in J309.
  - Will go over topics relevant to Pset.

Office hours from 5:30-6:30 in J517.

Look at course resources for Python and Math

Refresh Trigonometric and Complex number identities
  - \((x_1 + jy_1)(x_2 + jy_2) = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)\)
  - \(\cos(-\theta) = \cos \theta, \sin(-\theta) = -\sin \theta, \cos(\pi - \theta) = -\cos \theta, \ldots\)
Surface Normals

2-D

\[ \hat{n} \text{ Unit Vector Facing "outwards"} \]

\[ 90^\circ \]

Equation for a Line

\[ x \cos \theta + y \sin \theta = c \]

\[ \langle [x, y], [\cos \theta, \sin \theta] \rangle = c \]

\[ \langle [x, y], \hat{n} \rangle = c \]

Curve

\[ \hat{n}(x, y) \]

Defined at a point, Normal to the tangent at that point

\[ \hat{n} = [\hat{n}_x, \hat{n}_y] \]

\[ y = f(x) \quad x = g(y) \]

\[ \frac{\partial y}{\partial x} = -\frac{\hat{n}_x}{\hat{n}_y} \quad \frac{\partial x}{\partial y} = -\frac{\hat{n}_y}{\hat{n}_x} \]
Surface Normals

2-D

\( \hat{n} \)  Unit Vector  
Facing "outwards"

90°

Line

Curve

\( \hat{n}(x, y) \)

Tangent

Defined at a point,  
Normal to the tangent at that point

Equation for a Line

\[ \langle [x, y], \hat{n} \rangle = c \]

\( \hat{n} = [\hat{n}_x, \hat{n}_y] \)

\( y = f(x) \quad x = g(y) \)

\( \frac{\partial y}{\partial x} = -\frac{\hat{n}_x}{\hat{n}_y} \quad \frac{\partial x}{\partial y} = -\frac{\hat{n}_y}{\hat{n}_x} \)
Surface Normals

3-D

\( \hat{n} \) Unit Vector
Facing "outwards"

90°

Plane

Surface

\( \hat{n}(x, y, z) \)

Tangent Plane

Defined at a point,
Normal to the tangent plane at that point

Equation for a Plane

\[ \langle [x, y, z], \hat{n} \rangle = c \]

\( \hat{n} = [\hat{n}_x, \hat{n}_y, \hat{n}_z] \)

\( z = f(x, y) \)

\( \nabla z = \begin{bmatrix} \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \end{bmatrix} = \begin{bmatrix} \hat{n}_x & \hat{n}_y \\ \hat{n}_z & \hat{n}_z \end{bmatrix} \)
Surface Normals

3-D

\[ \mathbf{n} \]

\[ 90^\circ \]

\[ \hat{n} \]

\[ \mathbf{n} \]

Plane

Surface

\[ \hat{n}(x, y, z) \]

Tangent Plane

 Doesn't change if I scale entire surface

Defined at a point, Normal to the tangent plane at that point

Equation for a Plane

\[ \langle [x, y, z], \hat{n} \rangle = c \]

\[ \hat{n} = [\hat{n}_x, \hat{n}_y, \hat{n}_z] \]

\[ z = f(x, y) \]

\[ \nabla z = \left[ \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right] = \left[ \hat{n}_x, \hat{n}_y \right] \]
NORMALS

Normal Field
\[ \hat{n}(x, y, z) \quad \hat{n}(x, y) \]
Defined only on surface points
If only one \( z = f(x, y) \)

Gradient Field
\[ \nabla Z(x, y) = \left[ \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right](x, y) \]

Gradient / Normal fields are integrable, i.e., integrating along a closed curve gives 0.
Angle subtended by a curve on a point, is length of curve projected on unit circle
Angles subtended by a curve on a point, is length of curve projected on unit circle.

Angle between two lines is the angle subtended by any curve joining the two lines.

Measured in Radians.
Solid Angle

**Area** subtended by a surface on unit sphere.

Measured in steradians
Differential angle

Take an infinitesimal part of a curve, that can be assumed to be a line segment, and find the angle it subtends.
Differential angle

\[ d\theta = \frac{dL \cos \alpha}{r} \]

\( \alpha \) is the angle between the curve normal and line to the point.

Same length subtends greatest angle if aligned with normal.

Otherwise, "foreshortened"
Differential solid angle

\[ d\omega = \frac{dA \cos \alpha}{r^2} \]
Differential solid angle

\[ d\omega = \sin\theta \, d\theta \, d\phi \]

\[ \phi \in [0, 2\pi] \]

\[ \theta \in [0, \pi/2] \text{ or } [0, \pi] \]
Radiance \( L(\theta, \phi) \) is defined in terms of power \( P \) that the infinitesimal patch \( dA \) is pushing out in the infinitesimal solid angle \( d\omega \):

\[
L(\theta, \phi) = \frac{P}{(dA \cos \alpha)d\omega}
\]
Irradiance

How much light is arriving at a surface?
Irradiance

\[ \int L(\theta, \phi) \cos \theta d\omega = \int \int L(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \]

How much light is arriving at a surface?
Irradiance

\[ \int L(\theta, \phi) \cos \theta d\omega = \int \int L(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \]

What happens next? If this is a sensor, that's what it measures.
Bi-directional Reflectance Distribution Function

(Non illuminant) Surfaces will Absorb and Reflect portions of the incident light, in different directions.
Bi-directional Reflectance Distribution Function

(Non illuminant) Surfaces will Absorb and Reflect portions of the incident light, in different directions.

\[
\rho(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{L_o(\theta_o, \phi_o)}{L_i(\theta_i, \phi_i) \cos \theta_i d\omega_i}
\]
Bi-directional Reflectance Distribution Function

Total radiance in output direction from integrating contributions from all incoming radiance:

\[ L_o(\theta_o, \phi_o) = \int \rho(\theta_i, \phi_i, \theta_o, \phi_o)L_i(\theta_i, \phi_i) \cos \theta_i \, d\omega_i \]

- So, the BRDF describes how every incoming ray gets reflected by the surface.
  - How much energy in which direction
  - This is actually a function of wavelength \( \lambda \)
Bi-directional Reflectance Distribution Function

Properties

- Positivity: \( \rho(\theta_i, \phi_i, \theta_o, \phi_o) \geq 0 \)

- Helmholtz Reciprocity: \( \rho(\theta_i, \phi_i, \theta_o, \phi_o) = \rho(\theta_o, \phi_o, \theta_i, \phi_i) \)

- Total Energy leaving surface is less than total energy arriving

\[
\int L_i(\theta_i, \phi_i) d\omega_i \geq \int \left[ \int \rho(\theta_i, \phi_i, \theta_o, \phi_o) L_i(\theta_i, \phi_i) \cos \theta_i \ d\omega_i \right] \cos \theta_o d\omega_o
\]
RADIANCE

BRDF

Essentially a material property.

Outward Radiance in all directions same,
but still a function of input direction to normal.

Lambertian BRDF $\rho(\cdot) = K$

Mirror

Most of the radiance is in the mirror direction

Specular Highlights
BRDF

Essentially a material property.

Lambertian BRDF \[ \rho(\cdot) = K \]

The appearance won't change from change in viewing direction

Most of the radiance is in the mirror direction

Specular Highlights
Source: Matusik et al., A Data Driven Reflectance Model, TOG 2003
In all cases, reflected radiance depends on surface geometry, which we can exploit to estimate shape.