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Fairness: How to Measure It Quantitatively?

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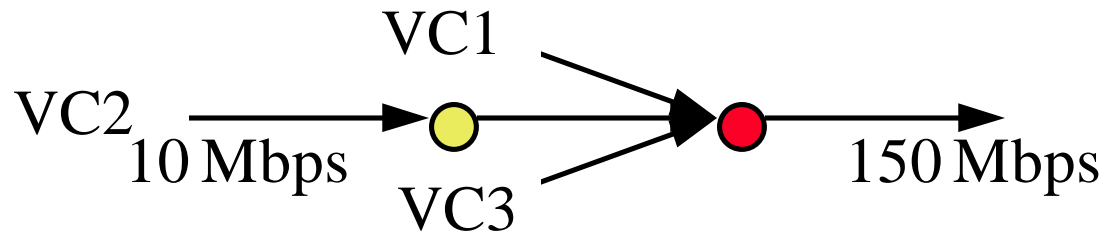


- Index of fairness
- Why is it better than others?

References:

- [1] R. Jain, W. Hawe, D. Chiu, "A Quantitative measure of fairness and discrimination for resource allocation in Shared Computer Systems," DEC-TR-301, **September 26, 1984.**
- [2] Raj Jain, "The Art of Computer Systems Performance Analysis," Wiley 1991

Fairness



- ❑ Simple Definition: **Equal** share of bottleneck
Problem: Some VC's may be bottlenecked elsewhere
- ❑ Next Definition: Optimal Allocation or Equal fraction of optimal allocation
- ❑ Example: A scheme gives 100, 4, 10.5 Mbps when the optimal is 100, 40, 15 Mbps
How fair is it? 67% ? 90% ?

Proposal

- ❑ Actual allocation: (A_1, A_2, \dots, A_n)
- ❑ Use any criterion (e.g., max-min optimality) to find the optimal allocation (O_1, O_2, \dots, O_n)
- ❑ Relative allocation: $x_i = A_i/O_i$

$$\text{Fairness} = \frac{(\sum x_i)^2}{n \sum x_i^2}$$

Example: $100/100, 4/40, 10.5/15 \Rightarrow 1, 0.1, 0.9$

$$\text{Fairness} = \frac{(1+0.1+0.9)^2}{3(1^2+0.1^2+0.9^2)} = \frac{2^2}{3(1+0.01+0.81)} = 0.73$$

Other Proposals

- Find the variance, standard deviation, or coefficient of variation

$$\text{Mean } \mu = (1+0.1+0.9)/3 = 0.67$$

$$\text{Variance } \sigma^2 = (1/n)\Sigma(x_i-\mu)^2 = 0.16$$

$$\text{Standard deviation } \sigma = 0.4$$

$$\text{Coefficient of variation} = \sigma/\mu = 0.597$$

- Find the distance from the optimal

$$\text{Fairness} = \frac{[\Sigma (A_i - O_i)^2]^{1/2}}{[\Sigma O_i^2]^{1/2}} = \frac{[0^2 + 0.9^2 + 0.1^2]^{1/2}}{[1^2 + 1^2 + 1^2]^{1/2}} = 0.52$$

- Min/Max = $0.1/1 = 0.1$

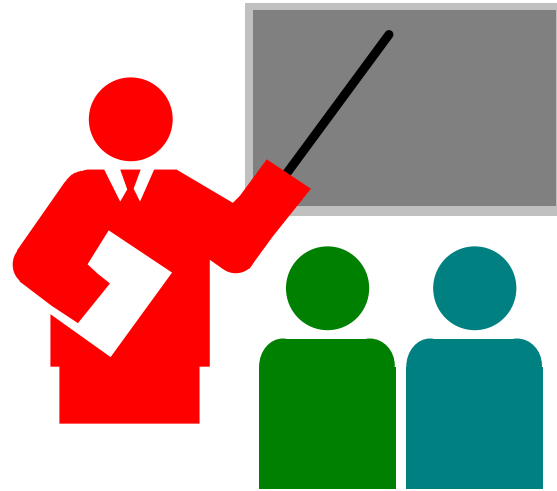
Fairness Index: Properties

- ❑ Applicable for any number of VCs, even $n=2$
Strictly speaking, variance not defined for small n .
- ❑ Scale independent.
Variance (Throughput) = $10 \text{ Mbps}^2 = 10^7 \text{ kbps}^2$
Standard deviation (Throughput) = $10 \text{ Mbps} = 10^4 \text{ kbps}$
- ❑ Bounded between 0 and 1 or 0 and 100%
Variance, standard deviation, and Relative distance are not bounded.
- ❑ Direct relationship: Higher index \Rightarrow More Fair
Higher variance \Rightarrow Less fair
- ❑ Continuous. Min/max is not continuous.

Fairness Index: Properties

- Intuitive:
 - For $(1, 0, 1)$ Index = $2/3$
 - For $x_i = 1, i=1,2,3,\dots,k$
= 0 otherwise
Index = k/n
 - If 80% of the users are treated fairly and 20% are starved, index = 80%

Summary



- The following text be added to the baseline text.

The fairness will be quantified using the following formula :

$$\text{Fairness} = \frac{(\sum x_i)^2}{n \sum x_i^2}$$

where x_i = ratio of actual throughput/optimal throughput.